

**Public Interest Energy Research (PIER) Program
FINAL PROJECT REPORT**

**IMPROVING METHODS FOR
ESTIMATING FATALITY OF
BIRDS AND BATS AT WIND
ENERGY FACILITIES**

Prepared for: California Energy Commission

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PREFACE

The California Energy Commission Public Interest Energy Research (PIER) Program supports public interest energy research and development that will help improve the quality of life in California by bringing environmentally safe, affordable, and reliable energy services and products to the marketplace.

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Improving Methods for Estimating Fatality of Birds and Bats at Wind Energy Facilities is the final report for the Energy Commission, Project Award Number PIR-08-028, conducted by California Wind Energy Association (CalWEA). The information from this project contributes to PIER's Energy-Related Environmental Research Program.

For more information about the PIER Program, please visit the Energy Commission's website at www.energy.ca.gov/research/ or contact the Energy Commission at 916-654-4878.

ABSTRACT

The California Wind Energy Association (CalWEA) evaluated the procedures in the *California Guidelines for Reducing Impacts to Birds and Bats from Wind Energy Development* (the Guidelines) for estimating fatality of birds and bats associated with wind energy facilities. The research sought to improve the accuracy of methods for estimating the number of bird and bat fatalities by evaluating the effect of time dependency on the probability of scavenging and removal of bird and bat carcasses (carcass persistence) and detection by searchers (searcher proficiency).

Researchers used data collected from the Altamont Pass Wind Resource Area from January 7 to April 30, 2011, to calculate traditional carcass persistence and searcher proficiency functions and to create new functions in which searcher proficiency and carcass persistence are modeled as a function of time and carcass age. This study is the first to document quantitatively that searcher proficiency and carcass persistence are time-based processes. The report offers lessons and implications for experimental designs and the field monitoring recommendations provided in the Guidelines.

The study also investigated the fatality estimation equation provided in the Guidelines and three other prominent equations from the literature that are used to adjust fatality observations for searcher proficiency and carcass persistence. The report examines both the common and equation-specific assumptions inherent in these fatality estimators, evaluates them in light of data from the field experiment, and finds that each of the fatality estimation equations can result in positive or negative bias, depending on the length of search interval relative to carcass persistence time. A new equation incorporating carcass persistence from one search interval to the next is proposed. This project will help reduce conflict in the siting process and make sound wind project permitting decisions easier by improving the accuracy of fatality estimates and the ability to accurately compare them with those from other wind facilities.

Keywords: Estimation methods, birds and bats, wind energy facilities, time dependence, searcher proficiency, carcass persistence, monitoring design, equations, statistical bias

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TABLE OF CONTENTS

PREFACE	i
ABSTRACT	ii
TABLE OF CONTENTS	iii
EXECUTIVE SUMMARY	1
Introduction.....	1
Purpose.....	1
Objectives and Findings.....	1
The Field Study: Design and Findings.....	2
Evaluation of the Fatality Estimation Equations.....	3
Conclusions and Recommendations.....	4
CHAPTER 1: RESEARCH PLAN	7
Statement of Need.....	7
Study Goal and Objectives.....	7
Success Measures.....	9
Statistical Considerations in the Experimental Design.....	9
Ecological Considerations in the Experimental Design.....	10
Ecological Variation.....	10
Size of the Carcasses.....	10
Scavenger Type and Density.....	10
CHAPTER 2: FIELD SAMPLING PROCEDURES AND RESULTS	11
Description of Study Area.....	11
Overview of Field Sampling Procedures.....	12
Specific Study Sampling Procedures.....	15
Carcass Placement.....	15
Carcass Searches.....	17
Carcass Status Checks.....	18
Quality Assurance/Quality Control.....	20
Results of the Field Sampling.....	20
Carcass Detections.....	20
Carcass Persistence Probability.....	23
Searcher Proficiency.....	26

CHAPTER 3: FATALITY ESTIMATION EQUATION ANALYSIS	29
Description of Analysis	29
The Estimating Equations	30
Common Assumptions.....	31
Erickson & Johnson’s Equation.....	32
A Variation: Shoenfeld’s Periodic Equation.....	34
Pollock’s Equation.....	36
Huso’s Equation	39
Discussion.....	40
The Figures	40
Comparing the Estimators.....	41
Variability	43
What if the Common Assumptions Fail?	43
Extensions.....	45
CHAPTER 4: STUDY FINDINGS AND RECOMMENDATIONS.....	46
Summary of Field Study Findings	46
Searcher Proficiency Shown to be Time-dependent	46
Searcher Proficiency is Site- and Species-specific	46
Searcher Proficiency is Lower for Bats than for Small Birds.....	46
Carcass Persistence is a Time-based Process.....	47
Summary of Estimating Equations Analysis	48
The Existing Estimators All Introduce Some Bias.....	48
Key Assumptions Distinguish the Estimators	48
Length of Search Interval Relative to Persistence Time is a Key Determinant of Bias	49
Conclusions and Recommendations.....	52
Current estimators do not account for time-dependent processes and “bleed-through.” ...	53
Care must be taken to avoid unacceptable bias when using current fatality estimators.	53
Use of previous study estimates.....	53
Future Research.....	54
CHAPTER 5: BIBLIOGRAPHY.....	55
APPENDIX A: A NEW EQUATION FOR ESTIMATING AVIAN MORTALITY RATES.. A-1	

APPENDIX B: A NEW SURVEY METHOD FOR DETECTION PROBABILITY TRIALS FOR PARTIAL PERIODIC ESTIMATOR.....B-1

 Introduction..... B-1

 Preliminary Scavenger Removal Trial B-1

 Main Study Detection Probability Trial B-1

 Carcass Placement..... B-1

 Integrated Carcass Persistence and Proficiency Detection Events..... B-2

 Search Interval Bleed-through of Carcasses: Theta..... B-2

 Collateral Data and Advantages to the New Method..... B-3

APPENDIX C: DATA DICTIONARY & DATA FIELDS USED FOR RECORDING CARCASSES..... C-1

APPENDIX D: FIGURES ILLUSTRATING BIASES FOR EQUATIONS IN THE CURRENT LITERATURE D-1

APPENDIX E: PUBLIC WEBINAR PRESENTATION E-1

EXECUTIVE SUMMARY

Introduction

Wind energy holds great promise as a clean, renewable energy resource, provided that siting and development can reasonably avoid or reduce impacts on already stressed wildlife resources. In 2007, the California Energy Commission and California Department of Fish and Game released *California Guidelines for Reducing Impacts to Birds and Bats from Wind Energy Development* (the Guidelines) to provide recommended procedures for assessing and minimizing impacts from wind energy development on birds and bats. The Guidelines provide an equation, attributed to Dr. Kenneth Pollock of North Carolina State University, that estimates the true number of fatalities at the wind facility from the number of bird or bat carcasses visually observed during a monitoring survey. The equation corrects for the inability of a searcher to locate all carcasses on the survey plot at the time of observation (searcher proficiency), and for the probability of removal by scavengers (such as crows and coyotes) or other processes before the time of observation (carcass persistence).

The California Wind Energy Association (CalWEA) rigorously evaluated the methods and procedures proposed by the Energy Commission for estimating the true number of fatalities of birds and bats (including the equation in the Guidelines) associated with collisions with wind turbines in California.

Purpose

This project sought to improve the accuracy of methods for estimating the number of bird and bat fatalities at wind energy facilities.

This report describes the sites selected for study, the experimental design for evaluating and testing approaches for estimating the true bird and bat fatalities at a wind facility from observational evidence of collision mortality, and the data collection procedures. This report also looks at the fatality estimation equation provided in the Guidelines and at three other prominent equations from the literature that are used to adjust mortality observations (hereafter referred to by their respective authors: Erickson & Johnson, Shoefeld, and Huso). It examines the assumptions common to all four estimation equations as well as those assumptions specific to each. It then evaluates the validity of the assumptions with data from the field experiment, given various field conditions, and fatality observation parameters. Based on the field study findings and a thorough analysis of assumptions underlying the published equations, this report offers lessons and implications for experimental designs and the field monitoring recommendations provided in the Guidelines.

Objectives and Findings

The project was designed to meet the following objectives:

- Refine and test experimental designs, under representative actual field conditions, that accurately generate site-specific data for estimating survey error rates.

- Rigorously evaluate the ability of various equations to accurately estimate fatalities of birds and bats at a variety of wind energy facilities within California.

The Field Study: Design and Findings

CalWEA rigorously designed and implemented a field survey to collect site-specific data under a variety of environmental conditions. Researchers obtained bird and bat carcasses from various labs and agencies and placed them at selected locations within the Altamont Pass Wind Resource Area near Livermore, California. Over periods of up to 60 days, independent and experienced biologists without prior knowledge of carcass placements searched strings of turbines weekly and recorded the location of marked bird and bat carcasses that project field managers had placed in the study area, as well as carcasses not associated with the study. Project field managers recorded the movement and removal of trial bird and bat carcasses roughly every three days during the study when trial birds and bats were on the ground, so that the true number and location of the trial carcasses were known. Consistent with current practice, it was assumed that carcass persistence and detection rates for marked carcasses placed at the site are representative of rates for bird and bat fatalities otherwise occurring at the wind energy facility.

Researchers used data generated by the field study to calculate traditional carcass persistence and searcher proficiency functions and to create new functions in which both carcass persistence and proficiency are modeled as a function of time and carcass age. Of the 104 small bird carcasses placed in the field, 32 unique carcasses (31 percent) were found over the course of 223 search opportunities (number of placed carcasses times the number of searches in which a trial carcass was present). However, field biologists detected carcasses in only 17 percent of all small bird search opportunities. Of the 78 bat carcasses placed, 15 unique bat carcasses (19 percent) were found over the course of 248 search opportunities, but only 8.1 percent of search opportunities yielded detections. All six of the large birds were detected, with 68 percent of 31 search opportunities yielding detections.

Researchers examined the rate of carcass removal by scavengers in strings (a group or row of adjacent wind turbines), blocks of strings with similar ecological conditions, and the entire study area. They also examined relationships between carcass persistence and key variables. The carcass removal rate followed a Weibull distribution, with the highest removal rates early in the trial. Scavengers removed most small birds and bat carcasses within six weeks of placement. The data also show that it was common for a carcass to persist into subsequent search intervals beyond the interval during which it was deposited (called “bleed-through”).

The study found both searcher proficiency and carcass persistence to depend on time. Other key findings with implications for selection of fatality-estimating equations and equation input variables include:

- Carcass persistence fits better with a Weibull distribution, where the attractiveness of a carcass to scavengers declines as it ages, than with an exponential distribution where fresh and old carcasses are equally likely to be attractive to scavengers.
- Vegetation height affects searcher proficiency. Therefore, when creating a survey design, researchers may want to consider random selection of turbines within blocks. The study found that topographical (for example, slope) and meteorological variables (for example, precipitation) were not correlated with mortality at the study site. They may be important predictors at other sites, however.
- Searcher proficiency was considerably lower for bats than for small birds during the study, pointing to the need for extensive long-term searcher proficiency trials for bats to ascertain if this holds true at other sites.
- Small bird carcasses are removed by scavengers more quickly than bat carcasses. This finding supports the need for long-term carcass persistence trials for both small birds and bats.

Evaluation of the Fatality Estimation Equations

As proposed, the second part of this project was to use the field study data to test how accurately the Pollock equation recommended in the Guidelines and the three other prominent equations estimate the true number of fatalities from observed fatalities. Because the equations assume that fatalities occur at random times, while this study involved placing all carcasses at the beginning of each experimental time block, a direct “test” of the equations using the study data was not appropriate. Instead, the authors analyzed the estimating equations (“estimators”) mathematically and tested the validity of their common and individual assumptions against the findings from the field study.

Key findings from this analysis were that:

- All of the four traditional fatality estimation equations examined assume constant searcher proficiency, rather than the observed condition that searcher proficiency is a function of time, as carcasses age. The inconsistent ability to detect a bird or bat over time can greatly affect the expected accuracy of resulting mortality estimates.
- Three of the equations examined (Erickson & Johnson, Shoenfeld, and Huso) assume an exponential distribution), whereas a Weibull statistical distribution fits the data best.
- Current estimators either assume that “bleed-through”– whether carcasses not removed during one search interval are considered “discoverable” during later searches – occurs all of the time or none of the time. Incorrect bleed-through assumptions can distort estimates.
- In the general case, and for exponential removal, the equations will generate mortality of the following order from lowest to highest: Erickson & Johnson < Shoenfeld < Pollock ≤

Huso. When choosing a single equation, investigators should keep the expected rank order in mind.

- The degree of systematic error or “bias” among the equations is a function of many issues, but in all cases, it is a function of the inherent assumptions underlying the equation characteristics. Even when biased, if search intervals are *long* relative to mean persistence times, all four estimators give about the same answers. But if search intervals are *short* relative to mean persistence times, large differences among the equations are possible. In fact, with the condition of short interval relative to mean carcass persistence, the results of the equations could differ by a factor of 3 or 4.
- Even correcting for the biases, the relationship of the results of the estimators to true mortality is unknown. However, if the assumptions in the equations are wrong (that is, where exponential distributions and constant searcher proficiencies have been assumed), then the results of the equations could differ significantly from actual mortality.
- Short search intervals increase the chance of bias:
 - (a) Short intervals do not allow the system to reach equilibrium, which is inconsistent with the Erickson & Johnson equation. Erickson & Johnson assume the number of carcasses remains relatively constant over the long-term.
 - (b) The Huso and Pollock equations assume zero percent bleed-through; therefore, bias will occur if true bleed-through is greater than zero.
 - (c) Shoenfeld assumes 100 percent bleed-through; therefore, bias will occur if true bleed-through is less than 100 percent.
- The new partially periodic equation proposed in this report allows for the estimation of a site-specific bleed-through rate. Paired with new field sampling procedures to generate time-dependent carcass persistence and searcher proficiency probabilities, this new equation will produce unbiased results using either short or long search intervals.

Conclusions and Recommendations

CalWEA’s study provides new insights that could enhance the existing methods and procedures found in the Guidelines and other pre- and post-construction fatality monitoring guidelines used in the United States and internationally. Four major implications of this work and the corresponding recommendations are outlined here.

- (1) Traditional fatality estimators do not account for time-dependence of carcass persistence and searcher proficiency, or for “bleed-through.”

Recommendation: Use the proposed new Partial Periodic Estimator and integrated detection probability trial method (proposed in Appendices A and B, respectively).

- (2) Traditional estimators can have high degrees of bias depending on the search interval, mean carcass persistence, and bleed-through rate of the field data collected.

Recommendation: Do not use traditional estimators in conditions that produce levels of bias that are unacceptable for the intended purpose. Caution is particularly warranted where short search intervals have been used.

- (3) Use of traditional estimators has resulted in an unknown degree of bias in the literature.

Recommendation: Carefully consider the value of metrics like “industry average” before applying them in policy or project-specific decisions.

- (4) Previously generated fatality estimates used for project evaluation or broader purposes could be recalculated using the proposed new Partial Periodic Estimator, provided the key input variables (search interval, mean carcass persistence, and so forth) can be collected from the original studies and reasonable assumptions made about searcher proficiency probability distributions and bleed-through values.

Recommendation: Going forward, use a standardized approach to generate unbiased, project-specific results that may be compared with each other, and to generate meaningful and unbiased industry averages and totals.

This project will help reduce conflict in the siting process and make sound wind project permitting decisions easier by:

- Providing guidance on methods for generating observer bias and carcass removal rates and reducing ambiguity in recommended avian study methods.
- Exploring time-dependent relationships, including observer bias and carcass removal.
- Providing guidance leading to improved field procedures for mortality monitoring and improving efficiency and efficacy of surveys.
- Enabling better forecasting of anticipated mortality at wind facilities based on site characteristics.

CHAPTER 1: Research Plan

Statement of Need

California pioneered large-scale wind energy development beginning in the 1970s. As a clean, renewable energy resource, wind energy holds great promise provided that it can be sited and developed in such a way as to reasonably avoid and (if necessary) mitigate impacts on already stressed wildlife resources. To this end, wind energy and wildlife stakeholders have collaborated to survey avian/bat activity and study the impacts of wind project operations, and policymakers have incorporated research protocols into the permitting process.

In 2007, the California Energy Commission (Energy Commission) and California Department of Fish and Game released *California Guidelines for Reducing Impacts to Birds and Bats from Wind Energy Development* (the Guidelines) to provide recommended protocols for assessing and minimizing impacts from wind energy development to birds and bats. The Guidelines recommend protocols for assessing, evaluating, and determining the effects of wind projects on birds and bats, and also recommend impact avoidance, minimization, and mitigation measures. In addition, the Guidelines provide an equation, suggested by Dr. Kenneth H. Pollock (personal communication, 2012), that can be used to adjust the number of bird or bat carcasses that are visually observed during an environmental monitoring survey of a wind facility, in an attempt to estimate the true fatalities at the wind facility. The equation, one of four analyzed in this report, adjusts for the inability of a searcher to locate all carcasses on the survey plot at the time of observation, and for the probability of removal by scavengers or other processes before the time of observation.

The California Wind Energy Association (CalWEA) received funding from the Energy Commission to rigorously evaluate the equations and associated procedures and studies recommended in the Guidelines for estimating fatalities of birds and bats associated with collisions with wind turbines in California. Information gathered from this study will apply to wind development projects in California, and the fundamental principles evaluated and discovered in this project may apply to wind development in other parts of the United States and internationally.

CalWEA's study provides new insights leading to improvements in the methods and procedures for estimating fatalities at wind facilities. This report offers recommendations on methods, including computations and data requirements, for estimating the true bird and bat fatalities at wind facilities. This section of the report details the goals of CalWEA's project and reviews statistical and ecological considerations in the project design.

Study Goal and Objectives

The overall goal of this project was to conduct research to improve the accuracy of methods for estimating the number of bird and bat fatalities at wind energy facilities. The project was designed to meet the following objectives:

1. Empirically test and calculate the influence of carcass removal and searcher proficiency under representative actual field conditions.
2. Mathematically evaluate the inherent characteristics and assumptions of existing equations to accurately estimate fatalities of birds and bats at representative wind energy facilities within California.

The study generated information to enable the evaluation of existing fatality estimation methods and the development of advanced models.

To meet the first project objective, CalWEA implemented a rigorously designed field survey at a wind facility within the Altamont Pass Wind Resources Area (APWRA) near Livermore, California. Site-specific data were collected under a variety of environmental conditions. Simply stated, birds and bats were placed at selected locations within the site. The implicit assumption in this approach is that marked birds and bats are representative of birds and bats killed at the wind facility. Over periods of up to 60 days, “blind” independent and experienced biologists without prior knowledge of carcass placements searched turbine strings and recorded the location of both marked bird and bat carcasses that project field managers had placed in the study area as well as carcasses not associated with the study.

Data generated during the experiment were collected and stored in a quality assured data set. The research team then analyzed the resulting data and evaluated the relationships among the number of found birds, bats, and environmental conditions over time. A description of the available statistical models evaluated in this study is found in the following discussion. The models and methods were evaluated for their inherent ability to accurately estimate the true number of bird and bat carcasses.

Once the study team evaluated the data, tested existing models and created new models, the team developed general guidance for (1) generating site-specific data used to parameterize equations, (2) selecting existing or new equations based on site-specific conditions, and (3) interpreting the results generated by the statistical methods.

This project provides insights into several other issues that are important to risk assessments of wind facilities. Specifically, this project generates information that can be used to:

- Evaluate existing fatality estimation methods.
- Test and evaluate the shape of carcass persistence curves (those not removed by scavenging, weather and other processes) under a variety of environmental conditions, as represented during the January – April grass height and weather conditions at the Altamont.
- Evaluate the effect of time-dependency on the probability of bird and bat carcass persistence and on the probability of detection by searchers (searcher proficiency).
- Develop recommendations for advanced models that link observational data with measurements of ecological conditions.

Success Measures

This project succeeded by achieving the following goals.

- Evaluation of the existing fatality equations provides practitioners information useful for choosing an estimating equation, and an understanding of the advantages and disadvantages of various equations with differing survey designs.
- Data generated from the project are of such quality that guidance for implementing site-specific studies leading to effective fatality adjustment procedures can be developed.
- Peer-reviewed publications can be generated that enhance the existing state of the science.
- Effective communication of the project findings was undertaken.
- Observational data at the planned wind turbine strings were obtained in a cost-effective manner within the timeframe of the project.

The following narrative discusses the statistical and ecological issues that influenced the design of the study, and presents the experimental design.

Statistical Considerations in the Experimental Design

Although standardized long-term monitoring procedures are available in the literature, there is currently no standard operating procedure for generating and evaluating data used to estimate fatalities at wind project sites. Statistical simulations of this issue have been conducted (*e.g.*, Huso 2010). In practice within the industry, searcher bias and scavenger removal studies are generally implemented in conjunction with long-term monitoring studies. However, based on an informal review and the experience of the authors of this report, there is little consistency in survey design and analysis of the resulting data among agencies, industry, or their consultants. Searcher bias studies are typically implemented independently from studies of removal by scavenging and other processes, and the study timeframes generally differ. In some cases, searcher bias studies are conducted once under site-specific conditions, and are not repeated during the course of a year. Carcass persistence studies are generally implemented over a few days to several weeks; however, the study time period is not standardized within the industry.

For both study types, fresh (or sometimes frozen) carcasses of various sizes are placed on an experimental plot at the beginning of the experiment. During searcher bias experiments, searchers search plots where trial carcasses have been placed and record the number of carcasses found. The searcher proficiency rate is then calculated and recorded. During scavenger removal studies, the known locations of the carcasses are observed frequently and removals are noted. Analysis of the resulting data generally provides a simple constant representing the probability that a bird or bat is removed by scavenging and other processes, although some time-series models resulting in the probability of scavenger removal as a function of time have been proposed (Smallwood 2007).

There is little consistency across searcher bias and scavenger removal studies in terms of plot area, number of carcasses used, carcass species, number of searchers tested, size of carcasses used, habitat considerations, or study timeframes. The relationship between searcher

proficiency and carcass persistence is not evaluated. Finally, the monitoring techniques employed during the searcher studies (*e.g.*, random searches, transect searches, search interval, etc.) are sometimes inconsistent with those employed during long-term site-specific monitoring studies at operating wind turbine facilities.

A number of equations are found in the peer-reviewed literature for adjusting the observable fatality counts to estimate the true number of killed birds and bats. This report reviews selected equations found in the literature, compares the properties of each of the estimators, and provides recommendations for improving their accuracy. The equations were chosen based on a review of literature that indicated that these equations have been commonly used within the wind industry. The equations are heavily cited in past and current peer-reviewed literature.

Ecological Considerations in the Experimental Design

The following discussion reviews the importance of key ecological variables in the estimation of survey error. In addition, key procedural and other experimental design variables are described.

Ecological Variation

Ecological variation associated with specific wind energy development sites within the State of California was an important consideration in the design of the experiments. Variation in habitat condition was considered a key variable affecting the change in survey error among locations. Variation in vegetation type and density, scavenger species and associated activity levels, climate conditions, geographic conditions associated with turbine placement, and a host of other site-specific variables also could influence the overall survey error rate for a specific site.

Size of the Carcasses

Carcass size is a key variable that influences both searcher detection proficiency and carcass persistence. Generally, larger birds (*e.g.*, golden eagles) are easier to see and are considered to have smaller survey error rates than smaller birds (or bats). The smaller birds (or bats) are more difficult to see over large distances, and may be more easily covered by vegetation. Also, smaller carcasses are more subject to removal by scavengers (see references found at http://www.altamontsrc.org/alt_rl.php).

The study's experiments were focused on smaller birds and bats based on the assumption that those carcasses are harder to find and therefore will have higher error rates. Carcasses representing similar size classes were used in the experiments.

Scavenger Type and Density

The activity level of scavengers at the test site(s) was an important consideration in the selection of the locations in which the experiments were conducted. Types of scavengers noted at the Altamont include birds (*e.g.*, ravens, crows, golden eagles, turkey vultures), and mammals (*e.g.*, foxes, coyotes, bobcats, raccoons, skunks, opossums, shrews, deer mice). Although scavenger activity was not monitored, the large number of scavenger species at the Altamont is expected to be representative of wind facilities across the United States.

CHAPTER 2: Field Sampling Procedures and Results

As discussed in detail below, field sampling involved marking bird and bat carcasses, placing them randomly at turbine strings at an operating wind farm, and collecting information on carcass persistence and searcher proficiency. Turbine strings were selected to represent varied environmental conditions, including vegetation type and height and slope.

Description of Study Area

The field study was conducted in NextEra Energy's Contra Costa County portion of the Altamont Pass Wind Resource Area (APWRA), which is located in north-central California approximately 56 miles (90.8 kilometers) east of San Francisco (Figure 1). Steady winds of 15–30 miles (25–45 kilometers) per hour blow across the APWRA during the mid-afternoon and evening periods between April and September, when 70–80 percent of the wind turbine power is generated in the APWRA (Smallwood and Thelander 2004).

The Altamont landscape consists of rolling hills ranging mostly between 150 and 300 feet (61–91 m) in elevation above sea level. Permits have been granted for a total of 5,400 wind turbines in the APWRA, rated at a capacity of approximately 580 megawatts (MW), distributed over 50,000 acres (150 square kilometers) of rolling grassland hills and valleys. Turbines are arrayed along ridgelines and other geographic features. The actual number of turbines available at any one time for power generation is thought to range from 4,500 to 5,000.

The APWRA supports a broad diversity of resident, migratory, and wintering bird species that regularly move through the wind turbine area (Orloff and Flannery 1996). Diurnal raptors (eagles and hawks), in particular, use the prevailing winds and updrafts for soaring and gliding during daily movement, foraging, and migration. Multiple studies of avian fatality at the APWRA show that golden eagles, red-tailed hawks, American kestrels, burrowing owls, barn owls, and a diverse mix of small birds and non-raptor species have been killed in turbine-related incidents (Howell and DiDonato 1991; Orloff and Flannery 1996; Howell 1997; Smallwood and Thelander 2004). All native species are protected by either federal and state wildlife legislation or both.

From an experimental perspective, the geographical unit of interest at the Altamont is a turbine string (a line of turbines). More than 400 of these strings have been monitored on a regular basis. The monitored strings are located over the extent of the APWRA, and therefore cover a

Figure 1: Location of Altamont Pass Wind Resource Area (APWRA)



Source: NextEra Energy Resources

variety of vegetation types and topological conditions.¹ Figure 2 shows the heterogeneity of the habitats around the field study wind turbines and strings.

Figure 2: Searching in Tall Grass and Short Grass



Photo Credit: EcoStat, Inc.

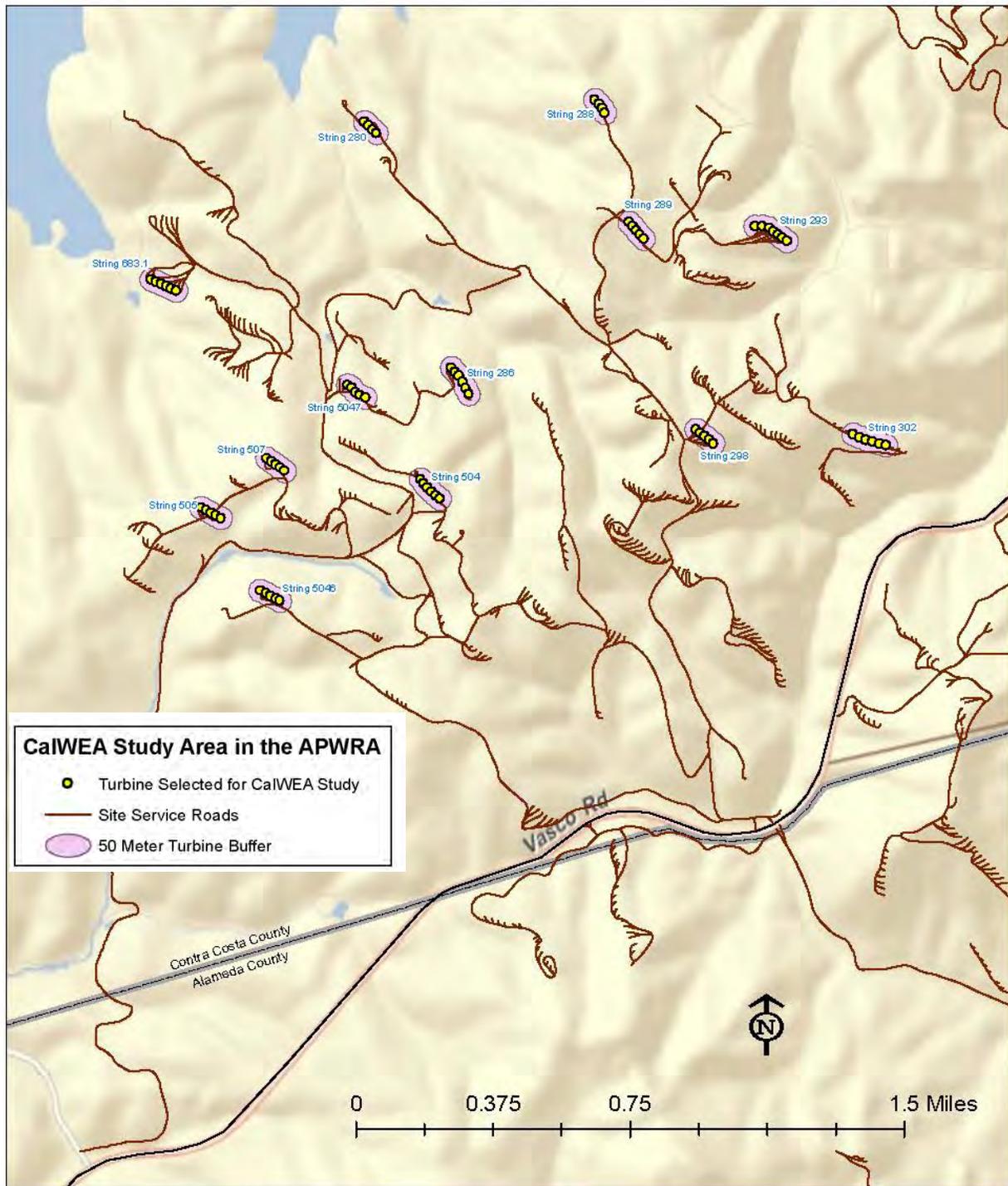
Overview of Field Sampling Procedures

Figure 3 shows the turbine strings where the field study was conducted. A total of 13 strings (four to seven turbines per string) were searched from January to April 2011. Strings were selected primarily so that search plots would not be mutually visible to ensure that searchers did not know the location of trial carcasses. Strings were also selected to represent the range of topological conditions and vegetation types in the Altamont. Strings were grouped into four blocks in which carcasses were concurrently placed and then monitored for four to six weeks. Blocks also served as a surrogate for vegetation and meteorological conditions over time. All strings monitored during the study's field trials as detailed below were located in the APWRA north of Vasco Road.

Before conducting the field study, a pilot study was conducted. This pilot study phase was used to test the work flow to fit the project resources and schedule and to test the field methods. The first block (Block 1) of the study area was used for the pilot study. Most of the same personnel were employed for block 1 as for other blocks. Block 1 was conducted at the same study site as the other blocks but with four strings instead of three. After the pilot study, the number of strings per block was set to three, and the number of placed trial carcasses was set to six bats and eight small birds per string.

¹ The natural communities and land cover types identified in the Natural Community Conservation Plan (NCCP) for the APWRA include agricultural land, annual grassland, alkali grassland, seasonal wetlands, alkali wetlands, perennial wetlands and ponds, riparian woodland and streams, chaparral, oak woodland, and conifer forest.

Figure 3: Field Study Wind Turbine Strings



Source: NextEra Energy Resources

Project field managers marked and randomly placed birds and bats and oversaw the recording of the carcasses and reporting of the data collected. The project field managers visited the strings every two days in order to verify the presence or absence of individual birds and bats. All birds and bats were uniquely marked, and any displacement of a bird or bat from the original location was observed and the new location noted for future reference. At the location of each bird or bat, project field managers took measurements of vegetation height. Project field manager observations provided an independent measure of the “true” number of birds and bats available for detection. Generally, six bats and eight birds were placed along each string. Halfway through the study, one large-sized bird was placed at each string within the block along with the standard six bats and eight small birds.

Once a week, a field technician searched an area around the study strings at a typical sampling walking pace, looking for any bird or bat carcasses.² On a typical day, a field technician conducted two string searches, averaging two to three hours per string, covering three to six acres. The field technicians were ignorant of the presence or absence of birds and bats at any specific string location. The field technicians recorded the position of observed carcasses. Project field manager status checks were timed to include checks on days that field technicians searched study strings in order to establish the true presence of carcasses available for detection by the field technicians. (To minimize false negative detections while maintaining field technician “blindness,” a cryptic system of marking carcass positions for project field managers was used.) Table 1 lists the field equipment used by the 11 field staff employed in the study.

Table 1: Equipment Used in the Field Study

Study Field Equipment	
4WD Trucks	Compasses
Clipboards	Cell phones
Data forms	Maps
Pen/Pencil/Sharpies	Hard hats
Camera/Scale card/Memory cards	Backpacks
Global Positioning System receivers (4m accuracy)	Yardsticks
Range finders	Markers (wooden stakes)

Source: EcoStat, Inc.

The Data Dictionary in Appendix C lists all the variables recorded, including weather information collected from January 1, 2011 through May 1, 2011 from the weather station at the Livermore, California, airport, and topographical variables recorded at each sampling location.

² Variable walking speed and direction across or along the ridge were not taken into account in this study, but would be interesting to consider in a future study.

Specific Study Sampling Procedures

Three procedures comprised the field study sampling methods:

- The placement of carcasses at study strings by project field managers.
- Blind carcass searches of study strings by field technicians.
- Status checks of placed carcasses at study strings by project field managers.

Carcass Placement

The purpose of the carcass placement procedure is to generate known random positions of marked carcasses at study strings.

Sources of Carcasses

Carcasses were provided by the following. For a variety of reasons, not all carcasses received were used during the field study.

- **Bat** carcasses: the Michigan Department of Community Health, Lansing, Michigan; Texas Christian University Department of Biology, Fort Worth, Texas; the Idaho State Department of Agriculture, Boise, Idaho.
- **Brown-headed cowbird** (*Molothrus ater*) carcasses: TW Biological Services, Fillmore, California; U.S. Department of Agriculture Animal and Plant Health Inspection Service/Wildlife Service, National Wildlife Research Center, Bismarck, North Dakota; Griffith Wildlife Biology, Calumet, Michigan.
- **Large bird** carcasses: Altamont Infrastructure Company, Livermore, California.

Carcass Position

The search area was defined by a 50 meter buffer created around turbines at study strings. A grid of 10-meter by 10-meter cells was projected over this search area. Topographical information was recorded for each cell (see Data Dictionary, Appendix C).

Grid cells were randomly selected for carcass placement. After grid cell selection, a project field manager would go to the approximate position of the selected grid cell and toss the marked carcass. The precise location of the carcass was recorded, including distance and bearing to the nearest turbine including the Global Positioning System (GPS) coordinates. In addition, the vegetation height immediately around the carcass position was measured. To help the project field managers find these selected carcass positions on future visits, a marker (small wooden stake) was cryptically placed 10 meters away from the carcass in such a way that a line segment was created by the position of the nearest turbine, carcass, and the marker.

Marked Carcasses

In order to maximize the project field managers' ability to identify individual trial carcasses, trial bird and bat carcasses were marked. Bird carcasses had a small amount of black tape attached to each leg marked with a unique obscured carcass identification number. In addition, the tips of the trial birds' flight feathers were cut. The tips of the trial bat carcasses' wings were taped and marked with a unique carcass identification number.

Table 2 shows the schedule for monitoring of the strings. The project field managers placed six bats and eight brown-headed cowbirds – referred to below as “small birds” – at each string, and placed one additional large bird at each string in Blocks 3 and 4. The goal was to run each block experiment for a six-week period, but logistical constraints sometimes shortened the time period, so that the actual durations ranged from 29 to 47 days. The first block experiment started on January 7, 2011, and the last block experiment ended on April 30, 2011.

Table 2: Summary of Sampling Design

Block #	String #	Turbine Address Range	Small bird carcasses placed at start of trial	Bat carcasses placed at start of trial ¹	Incidentally found carcasses added to study ²	Trial dates (2011)	Trial Length (days)
1	280	2206-2209	8	6	2	Jan 7-Feb 12	36
1	288	2038-2041	8	6	0	Jan 7-Feb 17	41
1	293	2075-2081	8	6	1	Jan 14-Feb 21	38
1	302	2166-2171	8	6	2	Jan 7-Feb 17	41
<i>Block 1 Subtotal</i>			32	24	5	Jan 7-Feb 21	45
2	298	2757-2761	8	6	1	Feb 18-Apr 4	45
2	683.1	2347-2354	8	6	10	Feb 18-Apr 4	45
2	5046	2542-2546	8	6	1	Feb 18-Mar 21	31
<i>Block 2 Subtotal</i>			24	18	12	Feb 18-Apr 4	45
3	286	2317-2322	9	6	2	Mar 11-Apr 22	42
3	289	2099-2103	9	6	0	Mar 11-Apr 22	42
3	507	2458-2463	9	6	0	Mar 11-Apr 27	47
<i>Block 3 Subtotal</i>			27 ³	18	2	Mar 11-Apr 27	47
4	504	2418-2423	9 ⁴	6	0	Apr 1-30	29
4	505	2514-2518	9 ⁵	6	0	Apr 1-30	29
4	5047	2377-2381	9 ⁴	6	2	Apr 1-30	29
<i>Block 4 Subtotal</i>			27	18	2	Apr 1-30	29
TOTAL, All Blocks			90	78	21	Jan 7-Apr 30	113

1. Species included big brown bats, little brown bats, silver-haired bats, unidentified *Pipistrellus*, and unidentified *Myotis* bats.
2. Mix of small and large birds (no bats), including some skeletal remains [note: evidence of skeletal remains are not used in the calculations presented in this report].
3. One complete red-tailed hawk carcass placed at each string in Block 3.
4. One complete common raven carcass placed at this string.
5. One complete California gull placed at this string.

Source: EcoStat, Inc.

Carcass Searches

The purpose of the carcass search procedure was to generate detection events of placed carcasses over time.

Field Technician Searches

Each study string was searched six times (once a week) over as many as six weeks. Each string search was conducted by one field technician who searched the entire 50-meter buffered search area using parallel transects, with an inter-transect distance of 6 to 8 meters depending on vegetation height and terrain (Figures 4, 5, and 6). Strict survey blindness was maintained by having each field technician search every study string only once over each six-week period, instructing the field technicians to not communicate found carcasses with each other, and to keep the number and position of marked carcasses a secret. Field technicians used range finders, compasses, and hand-held GPS receivers to navigate the search plots.

Figure 4: Conducting a Search



Photo Credit: EcoStat, Inc.

Figure 5: Searching in Short Grass



Photo Credit: EcoStat, Inc.

Figure 6: Searching in Tall Grass



Photo Credit: EcoStat, Inc.

In general, winter in the APWRA exhibits short vegetation starting to green due to rain. As temperature rises and precipitation continues, vegetation height increases and peaks in May.

Carcass Records

When a field technician detected a complete or partial carcass, or a collection of 10 or more feathers, a carcass record was created (Figures 7 and 8). In addition to placed marked carcasses, field technicians also found “natural” or incidental fatalities, which were also recorded. The Data Dictionary (Appendix C, Table C-1) lists all the variables contained in the final data set, including the data field technicians recorded when a carcass was found.

Figure 7: Fresh Bird Carcass



Photo Credit: EcoStat, Inc.

Figure 8: Partially Removed Carcass



Photo Credit: EcoStat, Inc.

Carcass Status Checks

The purpose of the carcass status check procedure is to rigorously verify the true status (presence, position, and condition) of known marked carcasses, both placed and incidentally found, at study strings.

Status Checks

Project field managers checked the status of all known carcasses every 48 hours and on days that field technician searches occurred (Figure 9). A project field manager found the last known location of a carcass utilizing a range finder, a compass, a GPS receiver, and a carcass marker. If an unknown carcass was found during a status check, the project field manager would collect and record data on its position and condition. (See Appendix C for complete list of data recorded for unknown carcasses.)

Project Field Manager Detection Types

Project field managers used range finders, compasses, and GPS receivers to find the approximate location of a placed carcass. If the carcass was not immediately detected, the carcass marker was sought out. The marker and turbine indicated a more precise carcass position. If the carcass was still not found, the position, the marker and turbine address became the point of origin for an intensive survey around this carcass to investigate if the carcass had

been moved by scavengers, degraded due to abiotic weathering processes such as wind and rain, or was removed. This intensive survey was typically conducted as a flushing search, a tight spiral transect out to 20 meters from the assumed last carcass position and again back to the point of origin reversing direction to maximize the view shed around obstructions such as high vegetation and rocks.

In addition to finding a placed carcass by its GPS position, marker, or a flushing search, new carcasses or carcass positions were found incidentally when project field managers walked between carcass positions or by field technicians during their carcass searches.

Figure 9: Project Field Manager Conducting a Status Check



Photo Credit: EcoStat, Inc.

Fractured Position, New, and Unknown Carcass Identification Numbers

Carcass scavenging sometimes fractured the carcass sign into multiple positions. If these carcass parts were distinct and more than 10 meters away from the initial carcass position, an additional carcass position was established and identified by a lettered suffix added to the carcass identification number (*e.g.*, 0121B). These newly established carcass positions were then checked along with other known carcass positions.

Occasionally new fatalities were found by field technicians during carcass searches or when Project field managers conducted status checks. These new carcasses were identified with a carcass identification number including the string number, the letter U, and the number of new fatalities found at that string (*e.g.*, 302U-01). These new fatalities were checked along with all other known carcass positions.

Sometimes a marked carcass was found but its carcass identification number was unknown because the identifying tape was missing due to scavenging actions. These unknown marked carcasses were identified with a carcass identification number including the string number, the letter M, and the number of marked carcasses found at that string (302M-01). These unknown marked carcass positions were checked along with all other known carcass positions. Later a

known carcass identification number was assigned to the carcass position based on its proximity to plausible known marked carcass positions.

Negative Detections

In order to maximize the certainty of a carcass position's removal, project field managers checked the negative presence (absence) multiple times before recording the removal of a carcass position. After a project field manager conducted three consecutive status checks, including flushing searches, with negative presence outcomes, the carcass position was declared removed and no longer part of future status checks. Once the carcass was confirmed removed, the time of removal was set consistent with the first observation time (this time is needed for the determination of the carcass persistence curve).

Quality Assurance/Quality Control

High frequency of data entry and field checks helped to assure the data was accurate:

- Data sheets from field technicians were collected after they completed their searches the same day and checked for completeness. The positions of any fatalities they found were also verified in the field on the same day by project field managers.
- Project field managers entered data into an Excel spreadsheet two to three times a week, because the data was needed to determine the status checks schedule.

If any questions arose when entering data, the data was rectified by asking the observer, using photos and GIS.

Results of the Field Sampling

Carcass Detections

Table 2 shows the number of trials in which a bird or bat carcass was truly on the ground, and a searcher had a chance of detecting the carcass. Carcasses that persisted over time contributed more to the number of trials than those that were removed from the study quickly.

Differences in the habitat types of the blocks may account for differences in carcass persistence, as well as the number of days on which a search occurred. Blocks are representative of changes in grass height over time; however, blocks were not selected based on specific ecological or habitat conditions. The chance of detecting a bird or bat was not equal for each search, and was found to be a function of vegetation height and carcass age. Topographical variables (*e.g.*, slope) and meteorological variables (*e.g.*, precipitation) were evaluated in addition to vegetation height, but were not found to be correlated to mortality at this site.

Table 3 summarizes the percentage of search opportunities with carcasses detected over the entire study. In practice, a single trial is implemented in which a fixed number of carcasses are observed. Each carcass has one chance of observation.

Table 2: Percent of Birds and Bats Observed for Each Block

Block	Species	Number of individual observations where the carcass was truly present ¹	Percent Detected	Average Vegetation Height (inches)
1	Bat	83	16.9	2.2
2	Bat	63	4.8	3.4
3	Bat	60	1.7	5.6
4	Bat	42	4.8	7.6
1	Small Bird	72	18.1	2.6
2	Small Bird	63	17.5	3.5
3	Small Bird	38	7.9	6.1
4	Small Bird	50	22.0	6.1
3	Large Bird	17	58.8	6.3
4	Large Bird	14	78.6	8.4

¹Note: individual carcasses could have several chances for observation during the study
Source: *EcoStat, Inc.*

Table 3: Percent of Birds and Bats Observed in Study

Species	Number of individual observations where the carcass was truly present ¹	Average Vegetation Height (inches)	Percent Detected
Bat	248	4.3	8.1
Small Birds	223	4.2	17.0
Large Birds	31	7.2	67.7

¹Note: individual carcasses could have several chances for observation during the study
Photo Credit: *EcoStat, Inc.*

Table 4 shows the chance that a carcass was observed on the first observation date. The number of bat carcasses observed on the first observation date is 14 percent. Note that the percentages observed on the first date are larger than found over all possible observation dates. This finding could be linked to increased difficulty with observing older carcasses.

Table 4: Percent of Birds and Bats Observed on First Observation Date

	Percent Detected
Bat	14.1
Small Birds	22.1
Large Birds	83.3

Source: EcoStat, Inc.

Table 5 shows average vegetation height by month and block. The vegetation in the study area is predominantly grass, with an average height of 2.7 inches (maximum 10 inches) at the start of the study in January and an average height of 6.4 inches (maximum 23 inches) at the end of the study in April.

Table 5: Average Vegetation Height (inches) Observed by Month and Block

	Jan 2011	Feb 2011	Mar 2011	Apr 2011
Block 1	2.7	2.3		
Block 2		2.3	3.2	4.3
Block 3			3.3	5.9
Block 4				6.4

Source: EcoStat, Inc.

Table 6 shows the number of individual carcasses detected for each block over the course of the entire study.

Table 6: Percent of Unique Carcasses Detected per Block (7-day interval)

Block #	Found			Placed			Percent Detected		
	Bats	Small Birds	Large Birds	Bats	Small Birds	Large Birds	Bats	Small Birds	Large Birds
1	10	11	-	24	32	-	41.7	40.6	-
2	3	8	-	18	24	-	16.7	29.2	-
3	1	3	3	18	24	3	5.6	12.5	100
4	1	10	3	18	24	3	5.6	37.5	100
Total	15	32	6	78	104	6	19.2	30.8	100

Source: EcoStat, Inc.

Carcass Persistence Probability

In this section, the scavenging rate at the level of string, block, and entire study area is examined. Relationships between carcass persistence and key covariates, such as vegetation height, are also examined. The persistence of a carcass on the field was modeled using a two-parameter Weibull distribution with the following model structure.

The density function for Weibull³ distributed carcass persistence times is the following:

$$p(t_i|\alpha, \lambda_i) = \alpha t_i^{\alpha-1} \exp(\lambda_i - \exp(\lambda_i) t_i^\alpha)$$

Where λ is the scale parameter, t_i is the time of event i , and α is the shape parameter of the Weibull probability density function.

The corresponding carcass persistence function can be written as follows:

$$S(t_i|\alpha, \lambda_i) = \exp(-\exp(\lambda_i) \lambda_i^\alpha t_i^\alpha)$$

Where S is the probability of carcass persistence (survival or non-removal from the field), and t_i is the time (days) that the carcass was observed on the field since the start of the study.

If covariates (*i.e.*, grass height, distance to bird or bat from the searcher, topographical features, etc.) are linked to λ with $\lambda_i = x_i' \beta$, where x_i is a vector of covariates corresponding to the i th observation (here, an observation is a survey date) and β is a vector of random parameters, the log-likelihood function is written as:

$$l(\alpha, \beta|t, x) = \sum_{i=1}^n v_i (\log(\alpha) + (\alpha - 1) \log(t_i) + x_i' \beta) - \exp(x_i' \beta) t_i^\alpha$$

The above model was implemented using a Bayesian paradigm with prior distributions:

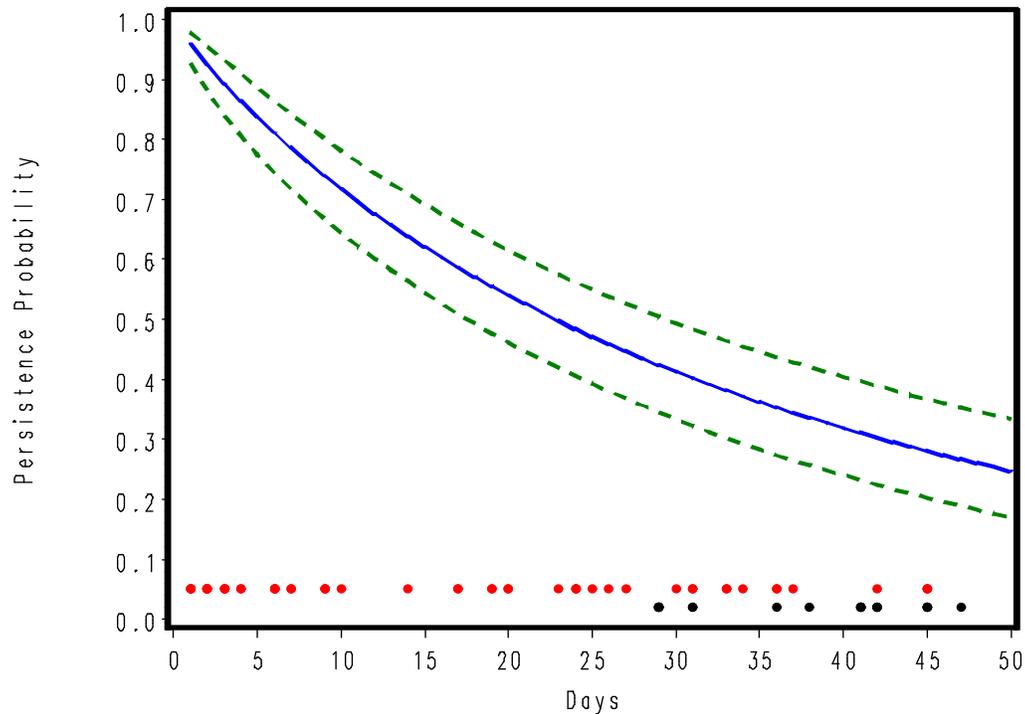
$$\beta: N(0, 10000)$$

$$\alpha: \text{Gamma}(0.001, 0.001)$$

Also, in some cases, the model was implemented without λ linked to covariates. Note that v indicates whether the observation is an actual failure time ($v=1$) or a censoring time ($v=0$). An observation is considered censored if the event of interest (in this case, the carcass is removed) does not occur within the timeframe of the study. A censored observation is defined as a record where the event (removal), has yet to occur (but, may occur if the record was tracked through time for a longer period). Results of the carcass persistence modeling exercise are shown below in Figures 10-13. These graphical presentations of the carcass persistence curves display the variability in probability within the data base. The curves are not adjusted for grass height, or other possible covariates.

³ The Weibull distribution is a continuous probability distribution used in survival analysis, which involves the modeling of time to event data.

Figure 10: Carcass Persistence Probability for All Bats in the Study



With 90 percent Credible Intervals, dashed line
Two-Parameter Weibull Survival Model

Red dots indicate a removal; Black dots indicate a censored⁴ record
Note: A single bat can be viewed more than once during the course of the study

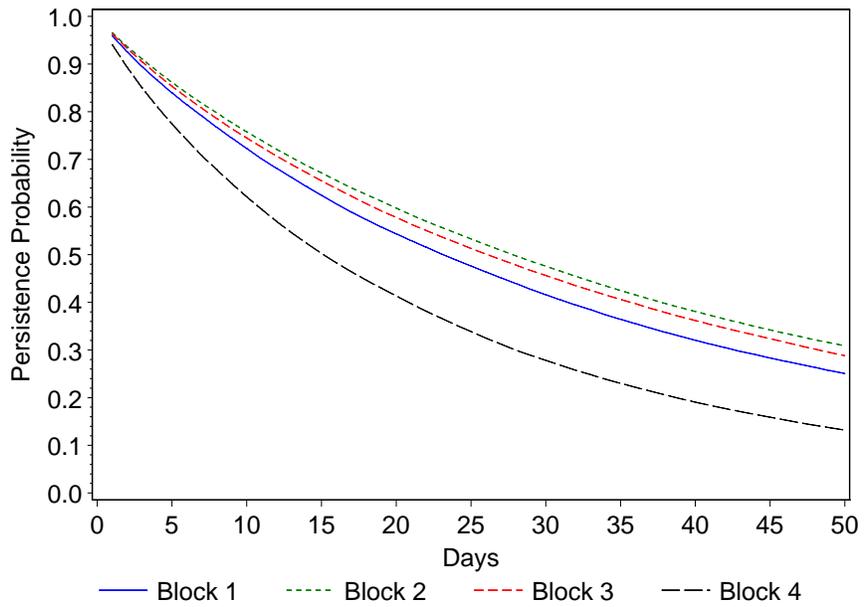
Source: EcoStat, Inc.

The carcass removal rate was high over the first two weeks and then the removal rate exponentially decreased. Red dots in Figure 10 indicate a constant rate of removal. Approximately 30 percent of bats were not removed (black dots).

Changes to grass height and other biological metrics over the study period may explain some of the differences in Figure 11. (However, no formal analysis of this subject is possible due to lack of rigorous field measurements). The statistical model does not result in a probability curve for large birds due to the low removal rate (one carcass).

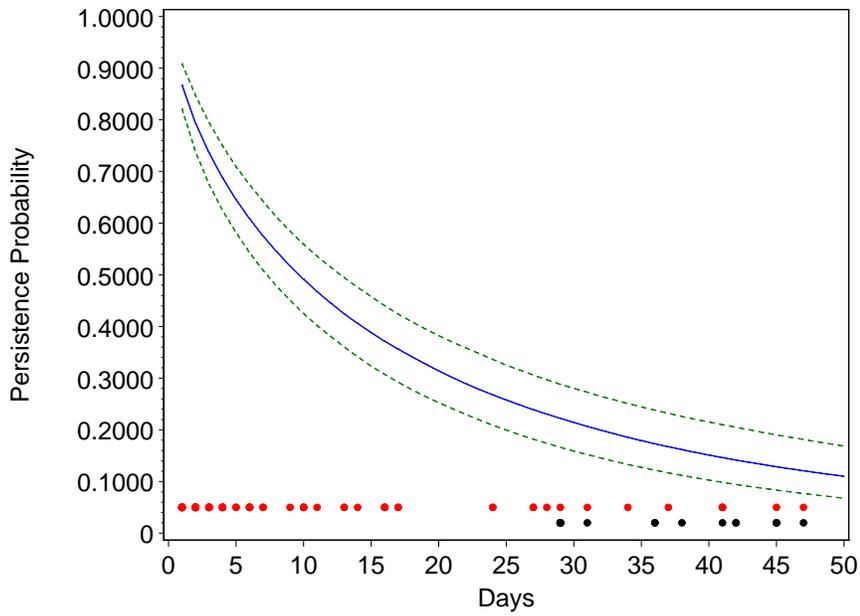
⁴ “Censored” means that the carcass remained on the ground (was not removed) when the trial ended.

Figure 11: Block-Specific Persistence Probability for All Bats in the Study



Source: EcoStat, Inc.

Figure 12: Persistence Probability for Small Birds in the Study

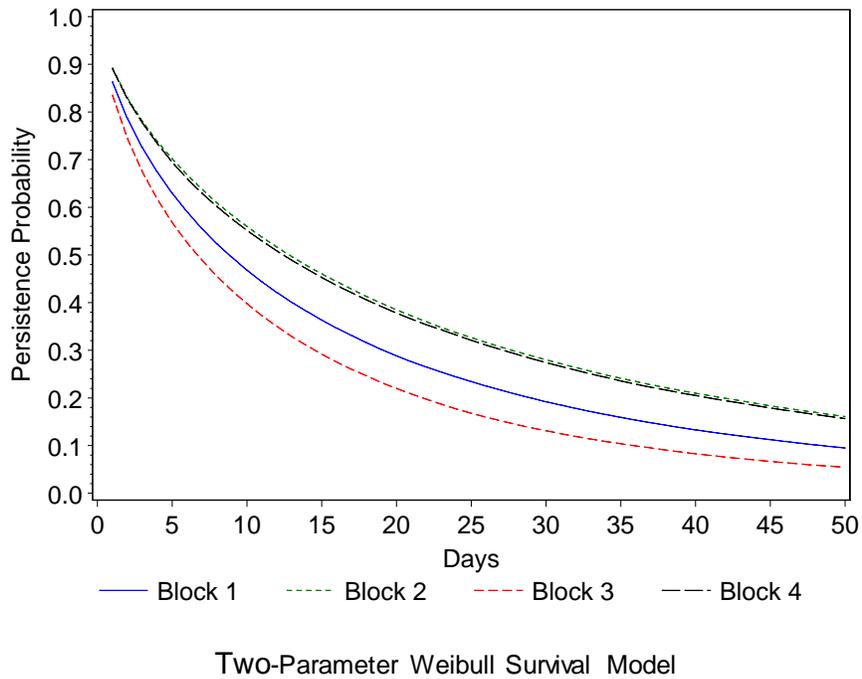


Dashed lines show 90 percent credible intervals

Red dots indicate a removal; Black dots indicate a censored record

Source: EcoStat, Inc.

Figure 13: Block Specific Persistence Probability for Small Birds in the Study



Source: *EcoStat, Inc.*

These curves confirm that the rates of carcass removal were greater in the first two weeks, and that most carcasses were removed within six weeks.

Searcher Proficiency

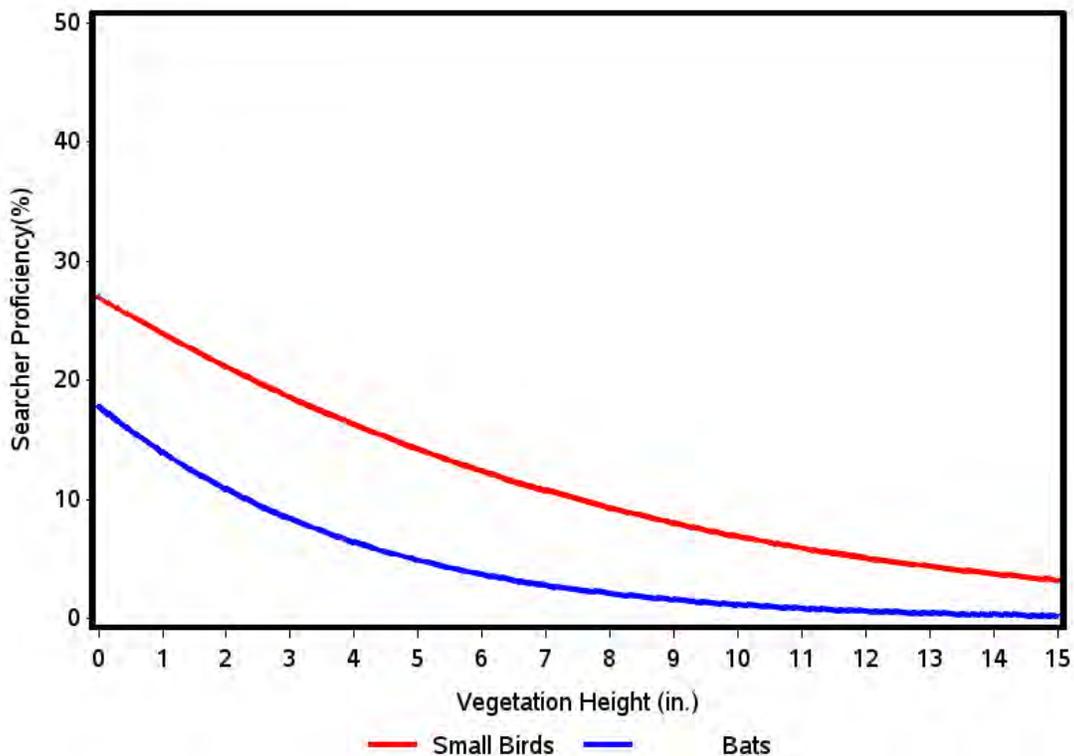
The magnitude of the searcher proficiency rate will be site specific, and will be a function of environmental and topological variables. In this study, searcher proficiency was significantly related to vegetation height (Figure 14). In addition to showing that searcher proficiency is a time-dependent process, Figures 14 and 15 clearly indicate that the shape of the searcher proficiency curves (with time and vegetation height) differ for birds and bats, and for small and large birds.

A key contribution of this study is the findings associated with bats. Statistics derived from this study indicate that, on average, searcher proficiency of bats is roughly half that of small birds. Large birds in this study were detected approximately 70 percent of the time. From a specific carcass perspective, approximately 30 percent of all small birds in the study were detected at least once, while only 19 percent of the bats were detected at least once.

The above rates for small birds are consistent with published literature values. For bats, however, the incorporation of time-based functions of searcher proficiency will have a significant impact on the resulting bat fatality estimation.

In this study, the searcher proficiency for small birds and bats was found to be similar after approximately 25 days, with the largest difference seen initially after carcass placement when the carcasses were fresh. An approximate 15 percent difference is seen between searcher proficiency in birds and bats with fresh carcasses. The searcher proficiency for birds and bats approached 2 percent after 30 days. This finding has implications for interval length in post-monitoring studies, where this study points to shorter intervals in order to maximize the chance of detecting a carcass on the ground.

Figure 14: Searcher Proficiency as Function of Vegetation Height for Brown-Headed Cowbirds and Bats, Integrated Across All Other Possible Covariates

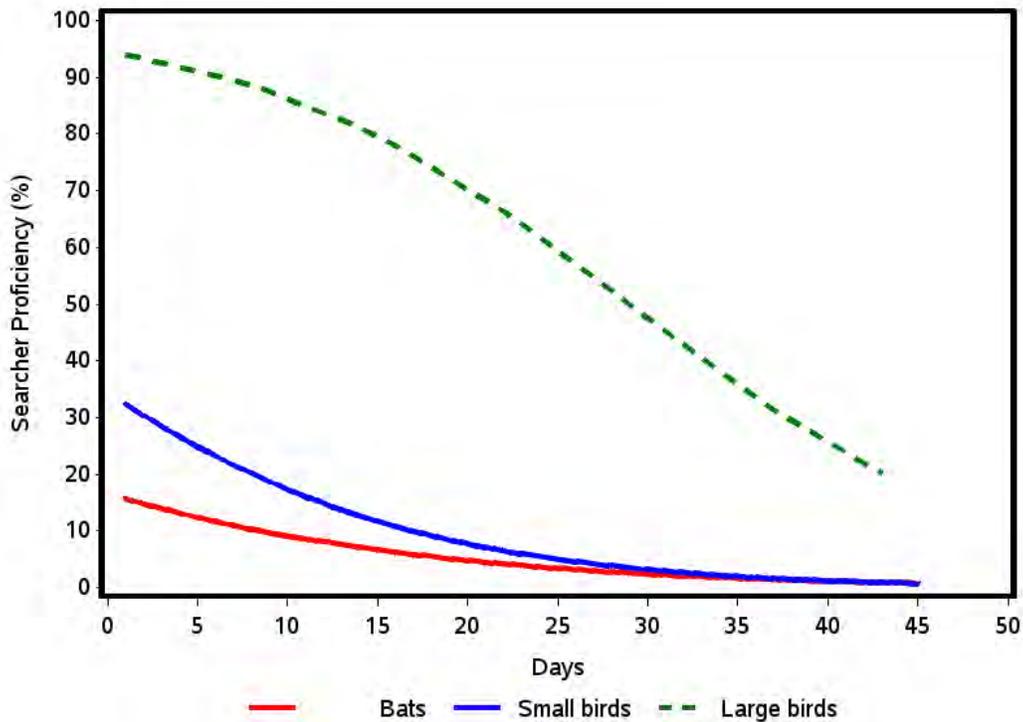


Source: EcoStat, Inc.

Bats are harder to find than birds, and all carcasses have low probability (less than 10 percent) of detection by field technicians after three weeks. The study’s finding that carcasses have the highest chance of being detected during the first two weeks has implications for study design. (Note that Figure 15 includes carcasses that have been scavenged but not removed.)

Table 7 presents the distance between the observed bird or bat, and the field technician. Statistics are calculated for the entire study, using all possible observations. Smaller carcasses are clearly shown to be found closer to the observer, on average. The distance sighted suggests that transects should be closer together; this study shows that 6 to 8 meters (a standard distance used by many investigators) is too far apart for many small bird and bat detections.

Figure 15: Searcher Proficiency of Small Birds and Bats Over Time, Integrated Over All Other Covariates



Source: EcoStat, Inc.

Table 7: Distance Between Observed Carcass and Field Technician

Species	Minimum Distance (meters)	Mean Distance (meters)	Maximum Distance (meters)
Bat	1.0	1.7	8.0
Small Birds	1.0	2.2	10.0
Large Birds	1.0	9.0	41.0

Source: EcoStat, Inc.

One problem with most estimators is that they must address a mix of species and ages of carcasses, which is complex. The time and age of carcasses matter for detection; the data reveal an often overlooked time dependency to searcher bias, combined with persistence.

Questions that could be explored with further research include whether increasing the searcher time per string (decreasing walking speed) results in higher detection rates, and whether it would be better to search one area thoroughly or search more areas.

CHAPTER 3: Fatality Estimation Equation Analysis

The objective of this section of the compare commonly used equations. Based on the assumptions underlying each equation, and the mathematics inherent in the equations, computer simulation is used to compare and contrast the expected true fatality rates among the equations evaluated. The equations are explored and evaluated using the concept of statistical bias and variance.

Description of Analysis

Estimating the true (or actual) fatalities of a specific species of bird or bat, related to a particular wind power generating facility during a specified time period, is a challenging task. Typical data supporting such estimates consist of collections $\{C_{ij}\}$ of counts of carcasses discovered by search teams in delineated search areas near a number of turbines (here indexed by i) at the end of successive search periods (here indexed by j), of varying length $\{I_{ij}\}$ (in days).

The simplest approach to estimating the total number M_{ij} of fatalities due to turbine i in time period j would be the raw count, $\widehat{M}_{ij} = C_{ij}$. This would be exactly correct under the simplistic assumptions:

- S₁ Each period begins with no carcasses in the search area;
- S₂ Each fatality caused by turbine j during period i leads to a (unique, single) carcass in the study area;
- S₃ There are no other sources of carcasses in the study area;
- S₄ Each carcass remains throughout the period;
- S₅ The search team discovers and removes every carcass.

Under these assumptions the total number M_{ij} of fatalities could be estimated perfectly by

$$\widehat{M}_{ij} := C_{ij}.^5$$

Each of the assumptions above is false to at least some degree, leading C_{ij} to be a badly distorted estimate of M_{ij} . Some of the reasons include:

- Experiments (for example, see http://www.altamontsrc.org/alt_rl.php) have shown that search teams usually discover only a fraction of existing carcasses (estimates ranging from 13 percent to 88 percent have been reported in the literature), violating S₅. The undiscovered carcasses will be present in the search area at the beginning of the subsequent period, violating S₁.
- Fatalities from turbine j may lead to carcasses outside the search area, violating S₂.

⁵ Note the equals sign (=) indicates “defined as.”

- Carcasses from fatalities caused by another turbine or from an unrelated source may fall into the search area, or carcasses from fatalities preceding the test period may persist into the period, violating S_3 .
- Scavengers may remove carcasses before they are discovered by the search team, or carcasses may degrade so much that they elude discovery, violating S_4 .

A number of authors have published more sophisticated estimation formulas for the number M_{ij} of birds or bats killed, intended to correct the biases induced by these issues. The following discussion is intended to explain the implicit assumptions that underlie four of these formulas, illustrating how they differ, and to offer suggestions for choosing among them or alternatives for the purpose of making reliable estimates of fatality.

The Estimating Equations

The authors study fatality by constructing a mathematical model in which the number C_{ij} of turbine-related carcasses discovered in the i^{th} spatial region at the end of the j^{th} temporal period is treated as a random variable. Each of the estimation formulas considered here begins as an equation expressing the *expected number* of carcasses counted, $E[C_{ij}]$, as a function of the actual number M_{ij} of fatalities and of some other factors (or estimates of them), under some assumptions about how scavenging and fatality proceed. This section considers what implicit assumptions lie behind these equations, offering some perspective on them and also some generalizations.

The authors differ in their choice of which letters to use as variable names for which quantities. To simplify comparing their estimation formulas, this report assigns common notation for all of them. Upper-case letters denote quantities which are (or could be, in principle) observed; lower-case letters denote model parameters. Table 8 presents the notation used here. “Hatted” quantities such as “ \widehat{M}_{ij} ” denote estimates of the corresponding quantities.

Even though observations are taken only at a few discrete times, it is useful to think of fatality and removal as processes that occur progressively over the time interval. Time is treated as a continuously-varying quantity t , measured in days, ranging from zero to I_{ij} during each study interval. The instantaneous rates of fatality and removal, and the levels of searcher proficiency, may vary in time and may depend on a variety of covariates. In a more detailed modeling effort the proficiency s_{ij} (the probability of discovery of a particular carcass) would depend on the searcher’s skill, the time lapse from fatality to search, and various covariates including the vegetation height and lighting conditions. Carcass removal rates r_{ij} would also change as carcasses age, and might depend on other covariates, leading to time and covariate dependence for persistence probabilities p_{ij} and average durations t_{ij} .

Table 8: Common Notation for Observable Quantities (Upper Case) and Parameter Values (Lower Case) for All Estimation Formulas

At turbine i in time interval j		
C_{ij}	(count)	= number of carcasses counted
I_{ij}	(search interval)	= search interval length (in days)
M_{ij}	(mortality)	= true number of carcasses during interval
p_{ij}	(persistence probability)	= probability a carcass remains unremoved until next search
r_{ij}	(removal rate)	= probability per day of carcass removal by scavengers and other processes
s_{ij}	(search proficiency)	= probability a carcass will be discovered
t_{ij}	(persistence time)	= average number of days a carcass remains unremoved

Source: Dr. Robert Wolpert

In this discussion, each of these parameters is treated as constant during each search interval, set to their *average* values in region i and epoch j . Models reflecting their dependence on time and covariates are under development and will be described elsewhere.

Common Assumptions

All four of the estimation equations below embody some common simplifying assumptions, most of them approximately correct or easily addressed:

- A₁: Each fatality caused by turbine j during period i leads to a carcass in the study area.
- In each of the approaches below this can be relaxed by including an additional factor $1/\hat{\pi}_{ij}$, where $\hat{\pi}_{ij}$ is an estimate of the fraction π_{ij} of carcasses from the j^{th} turbine that fall into the study area during the i^{th} time period. Most authors adjust for this.
- A₂: There are no other sources of carcasses in the study area.
- Searchers are trained to distinguish turbine fatalities from others, and search areas are sufficiently widely separated to ensure that few if any inappropriate carcasses will be counted.
- A₃: Carcass arrival times are uniformly distributed over the interval $[0, I_{ij}]$.

- Actual fatality rates will vary over time due to diurnal patterns, weather dependence, migratory patterns, and for other reasons, but the effects should average out over time with no significant effect on estimates.

A4: Quantities that vary over the time interval or that depend on covariates are adequately represented by their average values.

- This leads to considerable simplification, and holds approximately if the variation is small. See Discussion below for notes on how it may affect estimates if variation is not small, and on how it could be addressed.

Implicit assumptions specific to each particular estimation approach are described below.

Erickson & Johnson's Equation

An early attempt to reduce bias, attributed by Shoenfeld (2004, Equation (2)) to Erickson, Strickland, Johnson and Kern (1998) and by Huso (2011, §3.2) to Johnson, Erickson, Strickland, Shepherd, Shepherd and Sarappo (2003) is

$$\widehat{M}_{ij}^{EJ} = \frac{C_{ij}I_{ij}}{\hat{s}_{ij}\hat{t}_{ij}}. \quad (1)$$

If, on average, carcasses persist unremoved for only a fraction $t_{ij} < I_{ij}$ of the search interval, and if the search team's proficiency is $s_{ij} < 1$, it is reasonable to expect them to only discover a portion

$$C_{ij} \approx (t_{ij}/I_{ij})(s_{ij})M_{ij}$$

of the carcasses, leading to the estimator (1) when the uncertain quantities s_{ij} and t_{ij} are replaced with estimates and the equation is solved to construct an estimate of M_{ij} .

Exploring this in more detail, in the absence of intervention (*i.e.*, removal of carcasses by searchers) and under unchanging conditions, the long-term average number of carcasses present on the ground in the study area would reach a steady state with no systematic increase or decrease; denote the average number of carcasses at steady state by g_{ij}^{∞} . Since each of those carcasses is present for an average of t_{ij} days, the average daily fatality rate necessary to maintain that equilibrium is $m_{ij} = g_{ij}^{\infty}/t_{ij}$, so

$$g_{ij}^{\infty} = m_{ij}t_{ij}.$$

On average the total fatality in a period of I_{ij} days is $M_{ij} \approx m_{ij}I_{ij}$, so

$$g_{ij}^{\infty} \approx (M_{ij}/I_{ij})t_{ij}$$

and on average a search team that succeeds in discovering carcasses with probability $s_{ij} < 1$ (the team's proficiency) would discover a fraction s_{ij} of these,

$$E[C_{ij}] \equiv s_{ij}g_{ij}^{\infty} = M_{ij}s_{ij}t_{ij}/I_{ij}. \quad (2)$$

Replacing s_{ij} and t_{ij} by their estimates \hat{s}_{ij} and \hat{t}_{ij} and solving for M_{ij} leads to estimator (1), $\hat{M}_{ij}^{EJ} = (C_{ij}I_{ij})/(\hat{s}_{ij}\hat{t}_{ij})$. Because of its steady-state assumption, the validity of Erickson and Johnson's estimator \hat{M}_{ij}^{EJ} (1) requires the additional assumption:

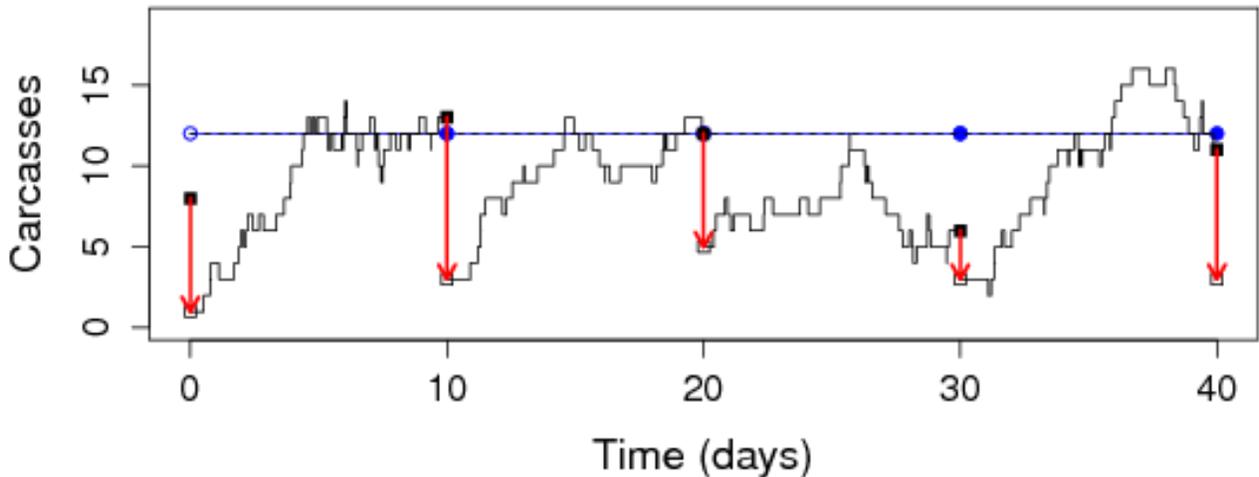
A_1^{EJ} : The system is in equilibrium at each search.

This will hold approximately whenever $I_{ij} \gg t_{ij}$, since the removal process then brings the system to equilibrium quickly, but in general it will be violated by any intervention such as the removal of discovered carcasses by search teams. If A_1^{EJ} fails (as in Figure 16) because of interventions that remove carcasses, then $C_{ij} < s_{ij}g_{ij}^\infty$ on average, leading to systematic underestimation with $\hat{M}_{ij}^{EJ} < M_{ij}$ (see Discussion below).

Figure 16 illustrates four $I_{ij} = 10$ -day periods. Simulated counts $G_{ij}(t)$ of carcasses currently in the study area are shown as a stair-step curve, for Poisson fatality at constant average daily rate $m_{ij} = 3d^{-1}$ and exponential persistence times averaging $t_{ij} = 4d$. The equilibrium average value $g_{ij}^\infty = m_{ij}t_{ij} = 12$ is shown as a horizontal line.

The curve $G_{ij}(t)$ increases by one with each new fatality (at random times chosen uniformly from each interval $[0, I_{ij}]$), decreases by one with each removal by scavengers (after independent exponentially-distributed persistence times), and decreases at the time of each search by the number of carcasses discovered and removed. Search team proficiency for the simulation is $s_{ij} = 0.70$. Search team carcass counts appear as downward arrows, and undiscovered carcasses remain for the subsequent search period.

Figure 16: Steady-State Value $g_{ij}^\infty = 12$ for Erickson & Johnson's Estimator \hat{M}_{ij}^{EJ} (1)



Horizontal line, beginning and ending each period at open and filled circles, respectively. One draw from random distribution (stair-step, beginning and ending each period at open and closed squares, respectively) is also shown, with discovered carcasses removed (in violation of A_1^{EJ}).

Source: Dr. Robert Wolpert

A Variation: Shoenfeld's Periodic Equation

Huso (2011, §3.2) attributes to Dr. Peter S. Shoenfeld (2004) the “modified” estimation formula

$$\widehat{M}_{ij}^S = \frac{C_{ij}I_{ij}}{\hat{s}_{ij}\hat{t}_{ij}} \left[\frac{e^{I_{ij}/\hat{t}_{ij}} - 1 + \hat{s}_{ij}}{e^{I_{ij}/\hat{t}_{ij}} - 1} \right] \quad (3)$$

Shoenfeld describes this estimator as a “periodic” variation on (1), specifically intended to address that estimator’s systematic underestimation, which he suggests is about 15–20 percent in practice. The next section reviews the assumptions implicit in Equation (3).

Each period begins with carcasses that were *not* discovered and removed by the previous search team still on the ground. As the number t of days into the period increases, the number of carcasses $G_{ij}(t)$ is increased by new fatalities and decreased by the removal process, with expected value $g_{ij}(t) = E[G_{ij}(t)]$ tending toward the equilibrium limit g_{ij}^∞ . Under the assumptions listed below, the mean satisfies a linear Ordinary Differential Equation:

$$\frac{d}{dt} g_{ij}(t) = m_{ij} - r_{ij}g_{ij}(t) = m_{ij} - g_{ij}(t)/t_{ij} \quad (4a)$$

where $m_{ij} = M_{ij}/I_{ij}$ is the daily fatality rate and $t_{ij} = 1/r_{ij}$ is the average persistence time. The well-known solution with initial value g_{ij}^0 is

$$g_{ij}(t) = g_{ij}^0 e^{-t/t_{ij}} + m_{ij}t_{ij}(1 - e^{-t/t_{ij}}), \quad (4b)$$

which begins at $g_{ij}(0) = g_{ij}^0$ and converges exponentially at rate $1/t_{ij}$ to the equilibrium value of $g_{ij}^\infty = m_{ij}t_{ij}$. The value at the time of the search ending the j^{th} time period is $g_{ij}(I_{ij})$.

Shoenfeld’s idea is to use this relation *periodically* for search scenarios where the search intervals, search proficiencies, and removal rates are approximately constant for consecutive time periods. In that case each period will end on average with the same number $g_i = g_{ij}(I_{ij})$ of carcasses as the preceding period. By periodicity, each must begin on average with $g_{ij}^0 = (1 - s_{ij})g_i$ carcasses, those undiscovered by the previous search team, leading to the equation

$$g_i = (1 - s_{ij})g_i e^{-I_{ij}/t_{ij}} + m_{ij}t_{ij}(1 - e^{-I_{ij}/t_{ij}}). \quad (5)$$

Collecting terms, this is easily solved for:

$$g_i = \frac{m_{ij}t_{ij}(1 - e^{-I_{ij}/t_{ij}})}{1 - (1 - s_{ij})e^{-I_{ij}/t_{ij}}} = \frac{M_{ij}t_{ij}}{I_{ij}} \frac{e^{I_{ij}/t_{ij}} - 1}{e^{I_{ij}/t_{ij}} - 1 + s_{ij}}$$

(using $m_{ij} = M_{ij}/I_{ij}$ for the average daily fatality). The expected carcass count will be less by a factor of the proficiency s_{ij} ,

$$E[C_{ij}] = M_{ij} \frac{s_{ij}t_{ij}}{I_{ij}} \left[\frac{e^{I_{ij}/t_{ij}} - 1}{e^{I_{ij}/t_{ij}} - 1 + s_{ij}} \right].$$

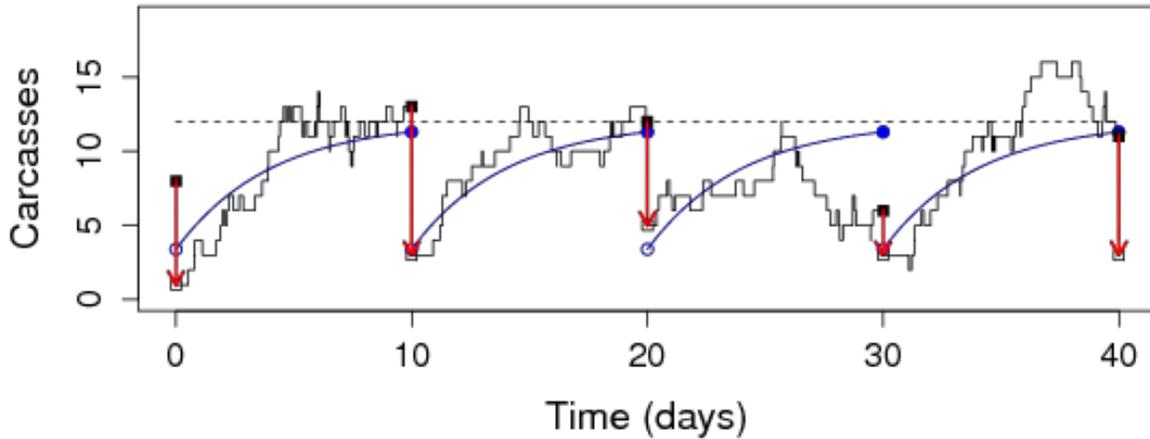
Solving for M_{ij} and replacing s_{ij} and t_{ij} with their estimates leads to Shoenfeld's (2004, Equation (1)) estimation equation,

$$\widehat{M}_{ij}^s = \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} \left[\frac{e^{I_{ij}/\widehat{t}_{ij}} - 1 + \widehat{s}_{ij}}{e^{I_{ij}/\widehat{t}_{ij}} - 1} \right].$$

Shoenfeld's periodic approach was based on three new assumptions (as inferred from the characteristics of the equation):

- A_1^s : Carcass persistence times have exponential distributions.
- A_2^s : All carcasses (both old and new) have the same probabilities of discovery s_{ij} .
- A_3^s : The lengths I_{ij} , rates of mortality m_{ij} and removal r_{ij} , and the proficiencies s_{ij} are approximately constant over consecutive time intervals.

Figure 17: Mean Function $g_{ij}(t)$ for Shoenfeld's "Periodic" Estimator \widehat{M}_{ij}^s (3)



Smooth solid curve, beginning and ending each period at open and filled circles, respectively

Steady-state limit (dashed curve at $g_{ij}^\infty = 12$), and one draw from random distribution (stair-step, beginning and ending each period at open and closed squares, respectively) are also shown. True mortality rate is $m_{ij} = 3d^{-1}$, persistence is $t_{ij} = 4d$, and searcher proficiency is $s_{ij} = 0.70$.

Source: Dr. Robert Wolpert

Assumption A_1^s was needed to justify the Ordinary Differential Equation (4). Assumption A_2^s ensures that undiscovered carcasses from an earlier period are just as likely to be removed by scavengers and weathering or discovered by future search teams as are fresh carcasses (see Discussion below), justifying their inclusion for the current period. Assumption A_3^s justifies the recursion of Equation (5).

If the sampling intervals I_{ij} are long compared to the average removal times t_{ij} , then the last factor in square brackets above is close to one and (3) reduces to (1), so $\widehat{M}_{ij}^s \approx \widehat{M}_{ij}^{EJ}$. If searches are

more frequent, so search intervals I_{ij} are not long compared to residence times t_{ij} , then the estimate \widehat{M}_{ij}^S of (3) always exceeds \widehat{M}_{ij}^{EJ} of (1), to compensate for the smaller number of carcasses on the ground following the previous search.

Figure 17 illustrates the model implicit in Shoenfeld's estimation equation for four $I_{ij} = 10$ -day periods. The mean value $g_{ij}(t)$ for the count $G_{ij}(t)$ of carcasses in the region is shown as a solid curve, beginning and ending each search period with an open or filled circle respectively, approaching but not quite attaining the steady-state $g_{ij}^\infty = 12$ shown as a dashed line. One random draw of the numbers $G_{ij}(t)$ of carcasses currently in the study area is shown as a stair-step curve for constant daily mortality rate $m_{ij} = 3d^{-1}$ and persistence times $t_{ij} = 4d$. Search team carcass counts appear as downward arrows; undiscovered carcasses remain for subsequent search period. As before, search team proficiency is $s_{ij} = 0.70$.

Pollock's Equation

It is worth questioning *why* in practice search teams find only a modest fraction s_{ij} of carcasses. Under Shoenfeld's assumption A_2^S the undiscovered carcasses are no harder or easier to find than those that were discovered — discovery failures are entirely random. But another possibility to consider is that some carcasses are more difficult to find than others, perhaps because they fell in deeper grass, or in an area with poorer light or less contrast, and that search teams find all of the most accessible carcasses. If so, then carcasses remaining on the ground after a search should *not* be included among those that might be found during subsequent periods. The next equations considered are based on an assumption that each period begins with no discoverable carcasses present.

The estimator recommended in Guidelines, suggested by Dr. Kenneth H. Pollock of North Carolina State University (2007), is

$$\widehat{M}_{ij}^P = \frac{C_{ij}}{\hat{p}_{ij}s_{ij}}. \quad (6)$$

This is the estimator one would derive from a model in which the expected carcass count for the j^{th} period could be expressed as the product $E[C_{ij}] = M_{ij}p_{ij}s_{ij}$ of the mortality count M_{ij} , reduced by the "persistence probability" p_{ij} and the searcher proficiency s_{ij} .

The difficulty in interpreting this equation and assessing its validity lies with interpreting the persistence probability parameter " p_{ij} ", described by this study as the "probability that a carcass persists and is observable until the next search" and by the Guidelines as the "probability that a carcass has not been removed in an interval." Because some carcasses appear much earlier in the interval than others, some will be subject to removal by scavengers and weathering for longer times than others and so some will face a higher probability of removal.

Exponential Persistence Times

If persistence times have *exponential* distributions, then the probability of persisting unremoved from any time $0 \leq t \leq I_{ij}$ to the end of the interval is $P[\tau_k > (I_{ij} - t)] = e^{-r_{ij}(I_{ij}-t)}$. Under

Common Assumption A₃ of uniformly-distributed arrival times, the average probability p_{ij} that a carcass persists until the next search at time I_{ij} and the average persistence time t_{ij} is given by

$$p_{ij} = \frac{1}{I_{ij}} \int_0^{I_{ij}} e^{-r_{ij}(I_{ij}-t)} dt = \frac{1}{r_{ij}I_{ij}} [1 - e^{-r_{ij}I_{ij}}] \quad (7a)$$

$$t_{ij} = \int_0^{\infty} e^{-r_{ij}\tau} d\tau = 1/r_{ij}. \quad (7b)$$

Combining these with (6), Pollock's estimator for exponential persistence is

$$\widehat{M}_{ij}^{P:E} = \frac{C_{ij}}{\widehat{p}_{ij}\widehat{s}_{ij}} = \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} [1 - e^{-I_{ij}/\widehat{t}_{ij}}]^{-1} \quad (\text{with exponential persistence}). \quad (7c)$$

Weibull Persistence Times

For exponentially-distributed persistence times, the probability of a carcass's removal during a day (assuming it is still present at the start of that day) does not vary with the age of the carcass. This feature of the distribution, sometimes called "memorylessness" and sometimes called "constant hazard", may not be realistic if older carcasses appear less attractive to scavengers. An alternative probability distribution commonly used to model failure times with decreasing hazard is the *Weibull* family.

Pollock's estimator \widehat{M}_{ij}^P of Equation (6) can be used with a Weibull probability distribution for persistence times exhibiting decreasing hazard, by introducing a new parameter $\alpha > 0$ (the Weibull "shape" parameter). The case $\alpha = 1$ reduces to the exponential distribution as before, but for $0 < \alpha < 1$ the hazard (*i.e.*, removal rate) falls off like the power $r_{ij} \propto \tau^{\alpha-1}$ with increasing persistence time τ . The persistence distribution is then given by

$$P[\tau > t] = e^{-(r_{ij}t)^\alpha}, \quad t > 0$$

with average persistence probability and average persistence time given by

$$p_{ij} = \frac{1}{I_{ij}} \int_0^{I_{ij}} e^{-[r_{ij}(I_{ij}-t)]^\alpha} dt = \frac{1}{r_{ij}I_{ij}} P\left(\frac{1}{\alpha}, \left[\Gamma\left(1 + \frac{1}{\alpha}\right)r_{ij}I_{ij}\right]^\alpha\right), \quad (8a)$$

$$t_{ij} = \int_0^{\infty} e^{-[r_{ij}\tau]^\alpha} d\tau = \Gamma\left(1 + \frac{1}{\alpha}\right)/r_{ij} \quad (8b)$$

where $\Gamma(a)$ and $P(a, x)$ denote the Gamma and incomplete Gamma functions, respectively (Abramowitz and Stegun, 1964, §6.5). The resulting estimator from (6) is

$$\widehat{M}_{ij}^{P:W} = \frac{C_{ij}}{\widehat{p}_{ij}\widehat{s}_{ij}} = \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} P\left(\frac{1}{\alpha}, \left[\Gamma\left(1 + \frac{1}{\alpha}\right)I_{ij}/\widehat{t}_{ij}\right]^\alpha\right)^{-1} \quad (\text{with Weibull persistence}), \quad (8c)$$

not much less tractable than the exponential version (7c).

Other interpretations of p_{ij} (for example, the probability a carcass present at the beginning of the interval will persist to the end) or other persistence distributions lead to different

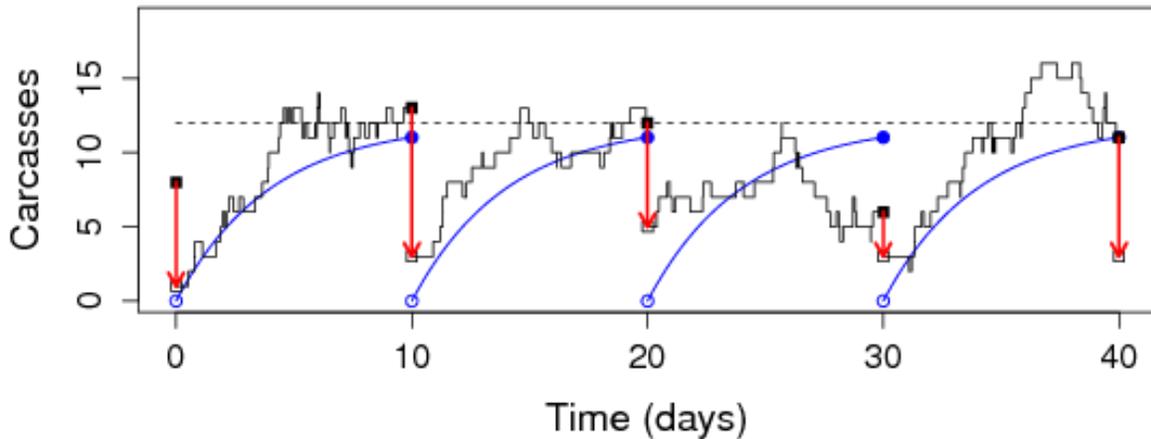
expressions and may require different assumptions for validity. For any persistence distribution, Pollock's estimator requires the assumption:

A_1^P : Each period begins with no discoverable carcasses.

If A_1^P fails then \widehat{M}_{ij}^P will consistently overestimate M_{ij} .

Figures 18 and 19 illustrate the model implicit for \widehat{M}_{ij}^P with exponential persistence for four $I_{ij} = 10$ -day periods. The mean value $g_{ij}(t)$ for the count $G_{ij}(t)$ of carcasses in the region is shown in each as a solid curve, beginning each search period with an open circle at $g_{ij}(t) = 0$ and ending each at a filled circle somewhat below the steady-state level of $g_{ij}^\infty = 12$, indicated by a dashed line. One random draw of the numbers $G_{ij}(t)$ of carcasses currently in the study area is shown as a stair-step curve for constant mortality rate $m_{ij} = 3d^{-1}$ and mean persistence times $t_{ij} = 4d$. Search team carcass counts appear as downward arrows, for proficiency is $s_{ij} = 0.70$. Following searches undiscovered carcasses remain discoverable for future searches in Figure 18, in violation of A_1^P , to illustrate possible bias, but search intervals are sufficiently long ($I_{ij} = 2.5t_{ij}$) that estimator \widehat{M}_{ij}^P has a bias of only 2.5 percent.

Figure 18: Mean Function $g_{ij}(t)$ for Pollock's Estimator \widehat{M}_{ij}^P (7c) with Exponential Carcass Persistence Distributions



Smooth solid curve, beginning each search period with an open circle at $g_{ij}(t) = 0$ and ending each period at a filled circle

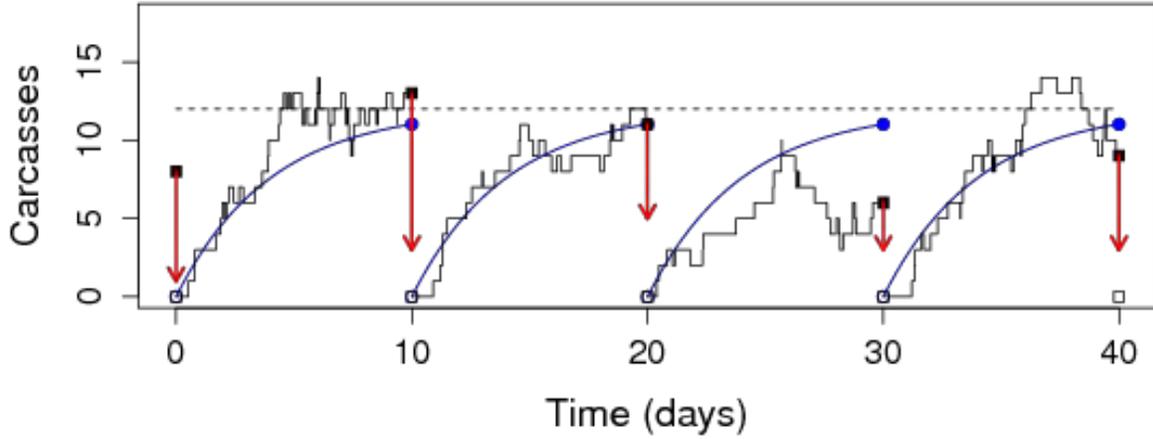
Steady state limit (dashed curve at $g_{ij}^\infty = 12$), and one draw from random distribution (stair-step, beginning and ending each period at open and closed squares, respectively) are also shown. True mortality rate is $m_{ij} \equiv 3d^{-1}$, persistence is $t_{ij} = 4d$, and search team proficiency is $s_{ij} = 0.70$.

Undiscovered carcasses are allowed to remain following searches, in violation of A_1^P .

Source: Dr. Robert Wolpert

For contrast, all carcasses are removed following searches in Figure 19, consistent with A_1^P .

Figure 19: Simulation Illustrating \hat{M}_{ij}^P (7c) with Exponential Persistence Distributions with Carcasses Removed Following Searches, so A_1^P Holds



Source: Dr. Robert Wolpert

Huso's Equation

Huso (2011) expresses the concern that in some study designs the interval I_{ij} between successive searches may far exceed the expected persistence time t_{ij} of carcasses. In that case she proposes to reduce the value used for I_{ij} to an "effective" time interval length $\tilde{I}_{ij} < I_{ij}$, sufficiently long that the random persistence times $\{\tau_k\}$ (with mean t_{ij}) will only exceed this effective time with small probability $P[\tau_k > \tilde{I}_{ij}] \leq 1 \text{ percent}$, and regard the carcass count as appropriate for just the last \tilde{I}_{ij} days of the interval. The resulting estimate is then scaled by the factor (I_{ij}/\tilde{I}_{ij}) to achieve an estimate \hat{M}_{ij}^H for the full interval of I_{ij} days. Under her assumption of exponential distributions for persistence times $\{\tau_k\}$, $\hat{I}_{ij} \equiv \hat{t}_{ij} \log(100)$ (about 4.6 times the estimated mean persistence time \hat{t}_{ij}), leading to Huso's estimator

$$\hat{M}_{ij}^H = \begin{cases} \frac{c_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij} (1 - e^{-I_{ij}/\hat{t}_{ij}})} & I_{ij} < \hat{I}_{ij} \\ \frac{c_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij} (0.99)} & I_{ij} > \hat{I}_{ij} \end{cases} = \frac{c_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij} [0.99 \wedge (1 - e^{-I_{ij}/\hat{t}_{ij}})]}. \quad (9)$$

This is expressed quite differently, but is mathematically identical to the "Proposed Estimator" of (Huso, 2011, §3.2, p.7). This estimate always exceeds Pollock's estimator $\hat{M}_{ij}^{P:E}$ (7c) for exponential persistence

$$\hat{M}_{ij}^H \geq \hat{M}_{ij}^{P:E} = \frac{c_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij} [1 - e^{-I_{ij}/\hat{t}_{ij}}]}. \quad (10)$$

The two never differ by more than one percent, and coincide whenever $I_{ij} < 4.6\hat{t}_{ij}$, so \hat{M}_{ij}^H may be viewed simply as a complicated way of expressing \hat{M}_{ij}^P for exponential persistence times. Huso's estimator will be valid and nearly unbiased under the assumptions:

- A_1^H : Each period begins with no discoverable carcasses.
- A_2^H : Carcass persistence times have exponential distributions.

The estimation equations considered here –

$$\widehat{M}_{ij}^{EJ} = \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} \quad (1)$$

$$\widehat{M}_{ij}^S = \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} \left[\frac{e^{I_{ij}/\widehat{t}_{ij}} - 1 + \widehat{s}_{ij}}{e^{I_{ij}/\widehat{t}_{ij}} - 1} \right] \quad (3)$$

$$\widehat{M}_{ij}^P = \frac{C_{ij}}{\widehat{p}_{ij}\widehat{s}_{ij}} \quad (6)$$

$$= \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} [1 - e^{-I_{ij}/\widehat{t}_{ij}}]^{-1} \quad \text{for exponential persistence} \quad (7c)$$

$$= \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} P \left(\frac{1}{\alpha}, \left[\Gamma \left(1 + \frac{1}{\alpha} \right) I_{ij}/\widehat{t}_{ij} \right]^\alpha \right)^{-1} \quad \text{for Weibull persistence} \quad (8c)$$

$$\widehat{M}_{ij}^H = \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij} [0.99 \Lambda (1 - e^{-I_{ij}/\widehat{t}_{ij}})]} \approx \frac{C_{ij}I_{ij}}{\widehat{s}_{ij}\widehat{t}_{ij}} [1 - e^{-I_{ij}/\widehat{t}_{ij}}]^{-1} \quad (9)$$

– are all intended to adjust for the gross underestimation of mortality M_{ij} by simple carcass counts C_{ij} . Each of them relies on the Common Assumptions A₁–A₄ (see p. 31) and each is a valid estimator of M_{ij} under some additional assumption (A₁^{EJ}, A₁^S – A₃^S, A₁^P and A₁^H & A₂^H, respectively).

Discussion

The Figures

Figures 16-19 illustrate the models for fatality and removal implicit in the estimators. Each figure shows simulated counts $G_{ij}(t)$ of carcasses in the area as solid black stair-step curves that increase by one with each new fatality, decrease by one with each scavenger removal, and decrease by C_{ij} at the end of the j^{th} interval upon the discovery and removal of C_{ij} carcasses by the search teams (each C_{ij} is indicated by a red downward arrow). In Figures 16–18, undiscovered carcasses remain present and may be discovered by later searches. To simplify comparison by focusing attention on what is different about the models (and not just random variation), the same fatality and removal times are used for each, so the functions $G(t)$ are identical in Figures 16-18. (In Figure 19, necessarily featuring different removal times, carcasses are removed following searches.)

The mean value functions $g_{ij}(t)$ implicit in the models are shown as solid blue curves, beginning each interval at an open circle and ending it at a filled circle (these overlap in Figure 16, where $g_{ij}(t)$ takes a constant value). Simulations and mean value calculations all use a daily fatality rate of $m_{ij} = 3\text{d}^{-1}$, so $10\text{d} \times 3\text{d}^{-1} = 30$ fatalities would be expected in each interval, or 120 overall (113 appeared in the simulation). Rate of removal by scavengers was $r_{ij} = 0.25\text{d}^{-1}$, so persistence times averaged $t_{ij} = 1/r_{ij} = 4\text{d}$ and, at steady-state, $m_{ij}t_{ij} = 3\text{d}^{-1} \times 4\text{d} = 12$ carcasses would be present. The search teams, whose proficiency was $s_{ij} = 70 \text{ percent}$, discovered 35 carcasses in the four intervals of the simulation.

Comparing the Estimators

The estimators fall into two groups. Estimators \widehat{M}_{ij}^{EJ} and \widehat{M}_{ij}^S each assume that some or all carcasses remain across searches, and that undiscovered carcasses from earlier time periods are removed or discovered at the same rates as fresh carcasses. In contrast, estimators \widehat{M}_{ij}^P and \widehat{M}_{ij}^H assume that each search period begins with no discoverable carcasses. For a fixed searcher proficiency and carcass persistence rate under the same site-specific characteristics, the ordering of the estimators is consistently:

$$\widehat{M}_{ij}^{EJ} < \widehat{M}_{ij}^S < \widehat{M}_{ij}^P \leq \widehat{M}_{ij}^H \quad (11)$$

for exponential persistence probability distributions. Note that then $\widehat{M}_{ij}^H \equiv \widehat{M}_{ij}^P$ unless $I_{ij} > 4.6\hat{t}_{ij}$, in which case they differ by at most 1.01 percent.

Choosing an Estimator

Which group of estimation equations is more appropriate for a particular species and experimental design — one of those (\widehat{M}_{ij}^{EJ} , \widehat{M}_{ij}^S) in which carcasses from earlier periods persist? Or one of those (like \widehat{M}_{ij}^P) in which each period is assumed to begin with no carcasses present?

Imagine sending two search teams with the same proficiency (say, 50 percent) into the same area in which, say, 20 carcasses are present, one after the other. The first team should find about 50 percent \times 20 = 10 carcasses, on average — but what would the *second* team find?

If they would be expected to find nothing, because all the discoverable carcasses would have been removed by the first team, then the Erickson & Johnson and Shoenfeld estimators (\widehat{M}_{ij}^{EJ} , \widehat{M}_{ij}^S) would not be appropriate. Both would underestimate M_{ij} by a factor of about $[1 - e^{-I_{ij}/\hat{t}_{ij}}]$, leading to a negative bias.

If they would find about 50 percent \times 10 = 5 carcasses (half those not found by the first team), then Pollock's and Huso's estimators would be inappropriate. Both would overestimate M_{ij} by a factor of about $[1 - (1 - \hat{s}_{ij})e^{-I_{ij}/\hat{t}_{ij}}]^{-1}$, leading to a positive bias.

Bias from Inappropriate Equation

These biases are apparent in the figures. In Figure 16, the stair-step simulated curves $G_{ij}(t)$ typically lie well below the Erickson & Johnson mean function $g_{ij}(t) \equiv g_{ij}^\infty$ and their endpoints (the filled squares) lie below g_{ij}^∞ on average, leading to underestimation (by -5.9 percent on average, for the parameters in this simulation). In Figure 18, the stair-step simulated curves typically lie above Pollock's mean function $g_{ij}(t)$ and the period endpoints, the filled squares, lie above $g_{ij}(t)$ on average, leading to overestimation (but only by +2.5 percent for the parameters used here). In Figure 17, the simulated curves $G_{ij}(t)$ coincide on average with Shoenfeld's mean function $g_{ij}(t)$, leading to accurate estimates. Figure 19 shows the degradation-based estimator $\widehat{M}_{ij}^P = \widehat{M}_{ij}^H$ with a simulation consistent with their assumptions (exponential persistence times and carcass removal following searches), so there is no bias.

The biases would be larger with more frequent searches, possibly considerably larger. Daily searches, for example, with the same residence time $t_{ij} = 4\text{d}$ and searcher proficiency $s_{ij} =$

70 percent, would lead to -71.1 percent bias for estimator \widehat{M}_{ij}^{EJ} and +30.5 percent bias for \widehat{M}_{ij}^H or \widehat{M}_{ij}^P , while \widehat{M}_{ij}^S would remain unbiased.

In the scenario of Figure 19, where undiscovered carcasses remain undiscoverable as if they were removed, Pollock's estimator (and Huso's which is identical) is unbiased while Shoenfeld's and Erickson/Johnson's underestimate M_{ij} by factors of $[1 - (1 - s_{ij})e^{-l_{ij}/t_{ij}}]^{-1}$ and $[1 - e^{-l_{ij}/t_{ij}}]$, respectively, for biases of -2.46 percent and -8.21 percent, respectively, with the 10-day search periods and 4-day persistence assumed here. For daily searches these biases would grow to -23.6 percent and -77.9 percent, respectively.

Bias Affecting All Equations

Each of the estimation formulas is based on an expression of the expected carcass count $E[C_{ij}]$ as a function of the fatality count M_{ij} and some other variables, such as the average persistence time t_{ij} and the search team's proficiency s_{ij} . An estimator is then constructed by solving this equation for M_{ij} as a function of $E[C_{ij}]$.

Consider, for example, estimator \widehat{M}_{ij}^{EJ} of Eqn. (1), derived from Equation (2), *i.e.*, the relation

$$E[C_{ij}]I_{ij} = M_{ij}s_{ij}t_{ij}.$$

If both s_{ij} and t_{ij} are uncertain or variable, perhaps because they depend on covariates (grass height, etc.) that themselves are variable or perhaps simply because they must be estimated from data, then there is still a linear relation for the expectations

$$E[C_{ij}I_{ij}] = E[M_{ij}\hat{s}_{ij}\hat{t}_{ij}]$$

for independent unbiased estimators \hat{s}_{ij} of s_{ij} and \hat{t}_{ij} of t_{ij} . Bias enters, however, when one makes the *non*-linear transformation of solving for M_{ij} :

$$M_{ij} \approx \widehat{M}_{ij} = \frac{C_{ij}I_{ij}}{\hat{s}_{ij}\hat{t}_{ij}}.$$

Because the function $x \rightsquigarrow 1/x$ is convex (its graph curves upward), the expectation of $1/\hat{s}_{ij}$ will *always* exceed $1/E[\hat{s}_{ij}]$, and that of $1/\hat{t}_{ij}$ will always exceed $1/E[\hat{t}_{ij}]$, so uncertainty or variability in s_{ij} and t_{ij} will lead each of these estimators to *overestimate* fatality to some extent, with $E[\widehat{M}_{ij}] > M_{ij}$. But how large is this positive bias?

If a positive random variable X has a log-normal distribution (commonly used to model uncertain positive quantities such as s_{ij} or t_{ij}) with mean $E[X]=M$ and variance $V[X]=V$, then $1/X$ also has a log-normal distribution, but the mean is *not* $1/M$. It is always larger:

$$E\frac{1}{X} = \frac{1}{M} \left[1 + \frac{V}{M^2} \right],$$

more than $1/M$ by a fraction V/M^2 .

Thus if \hat{s}_{ij} is an unbiased estimator of s with standard error ϵ , then $(1/\hat{s}_{ij})$ is a positively biased estimator of $(1/s_{ij})$ with bias given by:

$$E[1/\hat{s}_{ij}] = (1/\hat{s}_{ij}) \left[1 + (\epsilon/s_{ij})^2 \right]$$

with a similar formula for t_{ij} . If s_{ij} and t_{ij} are known to within a small proportional error, *i.e.*, if their standard errors are small fractions of their values, then little bias is introduced; if not, then more sophisticated statistical approaches may be warranted.

Variability

All the estimators considered here are of the form $\hat{M}_{ij} = \kappa C_{ij}$, proportional to the carcass count with a proportionality coefficient κ which will depend on I_{ij} , \hat{s}_{ij} , \hat{t}_{ij} , and perhaps other quantities. The value of κ is determined by solving for M_{ij} an equation for the expected number $E[C_{ij}] \propto M_{ij}$ of carcasses counted. The resulting *variability* of the estimators \hat{M}_{ij} can be quite large.

Because C_{ij} has a Poisson distribution under the models justifying all four of the estimators under consideration, and Poisson random variables have variance equal to their means, the variance of each such estimator $\hat{M}_{ij} = \kappa C_{ij}$ will be $\kappa^2 V[C_{ij}] = \kappa^2 E[C_{ij}] = \kappa E[\hat{M}_{ij}]$. Even an unbiased estimator \hat{M}_{ij} with expected value $E[\hat{M}_{ij}] = M_{ij}$ will have variance κM_{ij} that may be quite large. For counts high enough to justify a central limit approximation, one should expect typical estimation errors to be on the order of $\sqrt{\kappa M_{ij}}$,

$$|\hat{M}_{ij} - M_{ij}| \leq 1.645 \sqrt{\kappa M_{ij}} \approx 1.645 \sqrt{\kappa \hat{M}_{ij}}$$

with probability about 90 percent (and similar formulas for other quantiles). For counts C_{ij} too small to justify the central limit theorem, the Anscombe transformation

$$Z := 2 \sqrt{C_{ij} + 3/8} \sim No\left(2 \sqrt{c_{ij} + 1/8}, 1\right)$$

for $c_{ij} := E[C_{ij}]$ (Anscombe, 1948) leads to reliable interval estimates for M_{ij} for counts as low as $C_{ij} \geq 4$. Exact Poisson confidence intervals are available for all counts $C_{ij} \geq 0$.

For example, at the end of the second period of the simulation shown in Figure 17, $C_{i2} = 11$ carcasses were counted. With $s_{i2} = 0.7$ and $t_{i2} = 4$ estimated perfectly, Shoenfeld's estimator is $\hat{M}_{i2}^s = \kappa C_{i2}$ with

$$\kappa = \frac{I_{i2}}{\hat{s}_{i2} \hat{t}_{i2}} \left[\frac{e^{I_{i2}/\hat{t}_{i2}} - 1 + \hat{s}_{i2}}{e^{I_{i2}/\hat{t}_{i2}} - 1} \right] = \frac{10}{0.7 \times 4} \left[\frac{e^{2.5} - 1 + 0.7}{e^{2.5} - 1} \right] = 3.795$$

so a 90 percent Central Limit interval estimate is $\hat{M}_{i2}^s = 41.745 \pm 10.6 = [31.145, 52.345]$. The more accurate Anscombe approximation is $[24.21, 66.31]$ and the exact Poisson interval is $[23.41133, 69.09737]$. In the simulation $M_{i2} = 30$ fatalities occurred, exactly the expected number $I_{i2} m_{i2} = 10 \times 0.3 = 30$, but the 90 percent interval for this estimator ranges from -21.9 percent below the true value to +130.3 percent above it.

What if the Common Assumptions Fail?

Common Assumption A₁, that all fatalities lead to carcasses within the study area, is usually false because some carcasses may fall outside the designated study area, and some birds may be

crippled but able to make it outside the study region. If unaddressed, this “crippling bias” would lead to underestimation of fatality. It is usually addressed simply by estimating the probability π_{ij} that a fatality will lead to a carcass in the study area, then scaling any of the estimators ($\widehat{M}_{ij}^{EJ}, \widehat{M}_{ij}^S, \widehat{M}_{ij}^P, \widehat{M}_{ij}^H$) by a factor of $1/\hat{\pi}_{ij}$.

Common Assumption A₂ that all counted carcasses in the study region arise from encounters with the indicated turbine, is only approximately correct. Fatalities are usually assumed to be turbine related unless there is evidence to the contrary, but because the fatality rates from other causes are thought to be small enough this is not believed to lead to significant over-counting. A related problem is that some encounters with turbines may dismember a carcass into multiple pieces deposited in multiple discrete locations within the search area. Searchers attempt to prevent double-counting by matching parts, but this process is subject to error.

Search areas are generally established by rules of thumb, because of incomplete experimental data to suggest the true area of influence a turbine exerts, and may overlap. This could lead to misattribution, violating either A₁ or A₂.

Common Assumption A₃, that carcasses arrive uniformly over the time interval, will not be satisfied exactly. Actual fatality rates will vary over time with diurnal patterns, weather dependence, and other factors. If there are significant *trends* in fatality over the time period then this would affect each of the estimators, but haphazard variation on a rapid time-scale compared to search intervals will not. Some birds and bats have migratory behavior that may lead to widely differing rates from year to year or period to period, but if search intervals I_{ij} are short compared to migratory time scales then A₃ can still apply separately on each interval, but fatality and removal rates may vary for different time periods j .

Common Assumption A₄, that quantities are either constant or are sufficiently well represented by their averages, is also false. Both discovery by search teams and removal by scavengers are more difficult in areas or time periods within the study region where and when grass is taller, or light less available. Fortunately, these too are somewhat compensatory, but more elaborate modeling would be required to remove their effects entirely. Estimating s_{ij} and t_{ij} by imperfect estimators \hat{s}_{ij} and \hat{t}_{ij} does introduce some bias for all the estimators considered here, a rather technical issue.

Some estimators ($\widehat{M}_{ij}^S, \widehat{M}_{ij}^H$, and sometimes Pollock’s \widehat{M}_{ij}^P) also assume that carcass persistence times have exponential distributions. This distribution features a constant “hazard rate,” so its use implies that carcasses remain equally attractive to scavengers over time. Evidence suggests that this is false. Over time carcasses do deteriorate, with two effects: they become less attractive to scavengers, reducing the removal rate r_{ij} ; and they become more difficult for search teams to discover, reducing the proficiency s_{ij} . These two effects are somewhat compensatory, the first increasing and the second decreasing estimates of M_{ij} . If degradation is sudden and thorough enough it may be viewed simply as another form of removal by scavengers, maintaining validity for all the estimators, but if degradation is sufficient to deter scavengers but not enough to affect discovery that would lead to a positive bias.

Extensions

Each of the estimation approaches may be embellished to allow the rates of removal, fatality, or discovery to depend on meteorological, topographical, or other covariates, taken to be constant covariates for each turbine i and time interval j , at the cost of a considerable increase in computational complexity.

Coupled Degradation Models

In each of the models considered above the removal process and discovery are treated as “independent,” even for those underlying estimators \hat{M}_{ij}^P and \hat{M}_{ij}^H that feature degradation. If in fact carcasses differ in their appeal to scavengers and the ease with which they are detected by search teams, and if the same carcasses that are easy for search teams to discover are those that are rapidly removed by scavengers, then each of these estimators will be biased. Each on average will underestimate M_{ij} , because the easily discovered carcasses will have been removed preferentially. Equation (12) shows an extension of Pollock’s Weibull persistence equation (8c) for the most extreme case, where the removal and discovery processes are “coupled” in the sense that those carcasses with the longest persistence times are precisely those most difficult for search teams to discover:

$$\hat{M}_{ij}^x = \begin{cases} \frac{C I / \hat{t}_{ij}}{P\left(\frac{1}{\alpha}, \left[\Gamma\left(1 + \frac{1}{\alpha}\right) I / \hat{t}_{ij}\right]^\alpha\right) - (1 - \hat{s}_{ij}) I / \hat{t}_{ij}} & \hat{s}_{ij} > 1 - e^{(I/\hat{t}_{ij})^\alpha} \\ \frac{C I / \hat{t}_{ij}}{P\left(\frac{1}{\alpha}, (1 - \hat{s}_{ij})\right) - \alpha(1 - \hat{s}_{ij})[-\log(1 - \hat{s}_{ij})]^{1/\alpha}} & \hat{s}_{ij} \leq 1 - e^{(I/\hat{t}_{ij})^\alpha} \end{cases} \quad (12)$$

Intermediate cases between independence (8c) and coupling (12) are possible too. More details are presented in Appendix B along with a more elaborate model in which:

- Scavenger removal rates r_{ij} and search team discovery rates s_{ij} are allowed to depend on extrinsic covariates (grass height, for example) and on carcass age (hence persistence times will not have exponential distributions and counts may not be Poisson);
- Mortality rates m_{ij} need not be constant (seasonal and diurnal patterns may be explored),
- Hierarchical structure exploits the similarities expected for data from different but comparable time periods or search regions.

Each of the models underlying the estimators considered above can be expressed as a special case of that new model. Parameter estimation for the new model is more computationally intensive than the estimation formulas given here, however, and will require more extensive data collection, such as that described in Appendix B, which may not be available at all sites of interest.

CHAPTER 4: Study Findings and Recommendations

CalWEA's study offers several lessons with implications for the experimental designs and field monitoring recommendations provided in the Guidelines. The key findings, elaborated below, can be summarized under the following general statements:

- (1) Searcher proficiency is shown to be time-dependent.
- (2) Searcher proficiency is site- and species-specific.
- (3) Searcher proficiency is lower for bats than for birds.
- (4) Carcass persistence is a time-dependent process.
- (5) Small birds have a lower time-dependent persistence than bats.

In addition, CalWEA's analysis of the Guidelines' recommended fatality estimation equation (Pollock) and three other prominent estimators (Erickson & Johnson, Shoenfeld, and Huso) finds that:

- (6) All four of the equations reviewed introduce some bias.
- (7) The equations can be distinguished by their underlying assumption about whether undiscovered, unremoved carcasses remain "discoverable" in subsequent searches.
- (8) For all four equations, length of search interval relative to mean persistence time is a key determinant of bias.

These findings have implications for pre- and post-construction monitoring activities, discussed below along with a recommendation for development of an improved estimating equation that takes into account findings 6-8, above.

Summary of Field Study Findings

Searcher Proficiency Shown to be Time-dependent

This study is the first to document quantitatively the long-term relationship between carcass age and the ability to detect the carcass. The implications for this issue are large, and will influence survey methods, the number of carcasses used during detection trials, and the approach to conducting pre-survey detection trials.

Searcher Proficiency is Site- and Species-specific

The magnitude of the searcher proficiency rate will be site specific, and will be a function of environmental and topological variables. In this study, searcher proficiency was significantly related to vegetation height. In addition to showing that searcher proficiency is a time-dependent process, Figures 14 and 15 clearly indicate that the shape of the searcher proficiency curves (with time and vegetation height) differ for birds and bats, and for small and large birds.

Searcher Proficiency is Lower for Bats than for Small Birds

A key contribution of this study is the findings associated with bats. Statistics derived from this study indicate that, on average, searcher proficiency of bats is roughly half that of small birds.

Large birds in this study were detected approximately 70 percent of the time. From a specific carcass perspective, approximately 30 percent of all small birds in the study were detected at least once, while only 19 percent of the bats were detected at least once.

The above rates for small birds are consistent with published literature values. For bats, however, the incorporation of time-based functions of searcher proficiency will have a significant impact on the resulting bat fatality estimation.

In this study, the searcher proficiency for small birds and bats was found to be similar after approximately 25 days, with the largest difference seen initially after carcass placement when the carcasses were fresh. An approximate 15 percent difference is seen between searcher proficiency in birds and bats with fresh carcasses. The searcher proficiency for birds and bats approached 2 percent after 30 days. This finding has implications for interval length in post-monitoring studies, where this study points to shorter intervals in order to maximize the chance of detecting a carcass on the ground.

Carcass Persistence is a Time-based Process

For small birds, an initial 10-15 percent loss in total numbers can be expected in the first few days after first appearance. For bats, the initial loss rate is smaller, ranging from zero to approximately 6 percent. Again, this finding for bats may not be expected based on the current literature. In this study, the persistence probability for small birds was 50 percent at approximately 10 days, and less than 20 percent after 40 days. For bats, however, the persistence probability was approximately 50 percent at 25 days, and did not drop below 20 percent over the course of the study.

Carcass persistence curves can be a function of seasonal effects. Persistence curves for both small birds and bats differ over the course of the study timeframe.

Small birds have lower time-dependent persistence than bats

Based on this study, bats persist longer on the field than birds. While the relative time-process of persistence will be site-specific (at other sites the predator population may prefer bats), the finding of an increased persistence of bats relative to birds has implications for the ability of estimating equations to work well without a well-defined and rigorously tested persistence curve for bats. Coupling the longer persistence with the lower detection rates of bats as compared to birds could lead to gross error in the expected fatality of bats if new bat-specific estimating equations are not fully developed and tested. Indeed, because bats persist for relatively long periods and are difficult to see on the ground, the interaction of searcher bias and detection proficiency plays a significant role in accurately estimating bats. In particular for bats, long-term field trials rigorously designed to generate time-based searcher detection proficiency and carcass persistence rates will be critical to accurate estimation of bat fatality.

Carcass persistence is best fit with a Weibull distribution

The assumption of an exponential decay function in many existing equations was not directly tested in this study. A two-parameter Weibull function, which provides greater flexibility than the simple exponential assumptions, is shown to work well within the study conditions. As

noted in Chapter 3, the constant hazard assumption of the exponential function may not be realistic if older carcasses appear less attractive to scavengers, as shown in this study. The Weibull family of functions can be used to model carcass persistence without the assumption of constant hazard.

Summary of Estimating Equations Analysis

Existing fatality estimating equations assume that fatalities (and the corresponding occurrence of carcasses in a search plot) are randomly distributed over time. Because the experimental design of the CalWEA study did not allow for carcasses to be placed at random temporal intervals, direct calculation and comparison of the estimating equations against the known true number of birds and bats was not an appropriate test. Instead, equation properties and implicit assumptions were evaluated mathematically and the findings assessed in light of the findings from the field study.

The Existing Estimators All Introduce Some Bias

The CalWEA field study's finding that both searcher proficiency and carcass removal are time-dependent processes violates a common assumption of the four existing estimators that all carcasses are independent. This could easily be the case in this study where some carcasses specifically persisted and were not detected by the end of the study, indicating a lack of independence among the carcasses with respect to the two time-based processes.

If both searcher proficiency (s_{ij}) and mean persistence time (t_{ij}) are uncertain or variable, perhaps because they depend on covariates (grass height, etc.) that themselves are variable or perhaps simply because they must be estimated from data, then there is still a linear relation for the expectations for independent unbiased estimators \hat{s}_{ij} of s_{ij} and \hat{t}_{ij} of t_{ij} . Bias enters, however, when they are made the *non*-linear transformation of solving for M_{ij} .

Another common assumption, that quantities are either constant or are sufficiently well represented by their averages, is also false. Both discovery by search teams and removal by scavengers and weathering are more difficult in areas or time periods within the study region where and when grass is taller, or light less available. Fortunately, these too are somewhat compensatory, but more elaborate modeling would be required to remove their effects entirely. Estimating s_{ij} and t_{ij} by imperfect estimators \hat{s}_{ij} and \hat{t}_{ij} does introduce some bias for all the estimators considered here, a rather technical issue sketched in Chapter 3.

Key Assumptions Distinguish the Estimators

Each of the equations evaluated contains implicit assumptions pertaining to the nature of the rate of bird/bat fatality during the search interval, the distribution of carcass persistence times, and whether carcasses that persist from one search interval to the next are considered "discoverable" during a subsequent search. These distinguishing assumptions are summarized in Table 9.

Table 9: Key Assumptions Distinguishing Estimators Reviewed

Equation	Key Assumptions
Erickson & Johnson (1998)	A_1^{EJ} : The system is in equilibrium at each search. $I_{ij} \gg t_{ij}$
Shoenfeld (2004)	<p>A_1^S: Carcass persistence times have exponential distributions.</p> <p>A_2^S: All carcasses (both old and new) have the same probabilities of discovery s_{ij}. Undiscovered carcasses are no harder or easier to find than those that were discovered— <i>i.e.</i>, discovery failures are entirely random.</p> <p>A_3^S: The lengths I_{ij}, rates of mortality m_{ij} and removal r_{ij}, and the proficiencies s_{ij} are approximately constant over consecutive time intervals.</p>
Pollock (2007)	A_1^P : Each period begins with no discoverable carcasses
Huso (2011)	<p>A_1^H: Carcass persistence times have exponential distributions.</p> <p>A_2^H: Each period begins with no discoverable carcasses.</p>

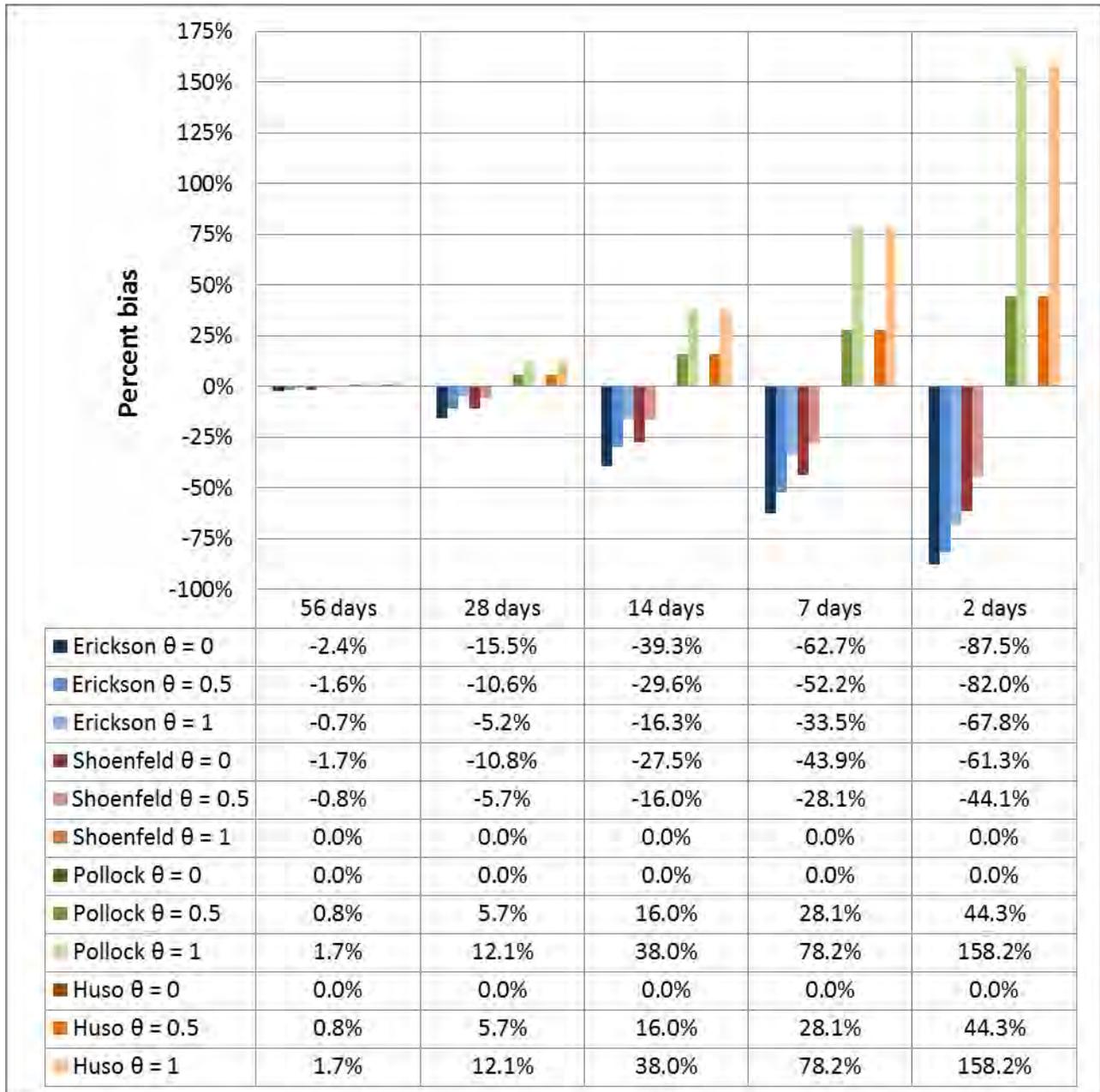
Source: Dr. Robert Wolpert

An important contribution of this analysis is the concept of “bleed-through” – the idea that every carcass not discovered and removed in a search, and does not persist due to scavenging, weathering, or other natural processes, remains for possible discovery in later searches. Both Erickson-Johnson and Shoenfeld’s estimators assume 100 percent bleed-through. Huso’s estimator assumes zero percent bleed-through – *none* of the carcasses not removed (by searchers or scavengers) are *ever* discovered in subsequent searches. Pollock’s estimator uses an “average probability a carcass is unremoved until the search” (p_{ij}) rather than the more commonly used “mean persistence time” (t_{ij}). But as with Huso’s estimator, Pollock’s implicit assumption is that each period begins with no discoverable carcasses (“old” carcasses are never discovered).

Length of Search Interval Relative to Persistence Time is a Key Determinant of Bias

When search intervals are long with respect to persistence times, the influence of this “carcass at the beginning of the search interval” assumption is minimized and the estimators are nearly unbiased and provide very similar answers. However, for very short search intervals (a growing tendency in the wind industry), the bias in some equations can be large, and the equations can provide very different results. Figures 20-21 illustrate this point, showing the range of bias in fatality estimates obtained using the various estimators with different search intervals and bleed-through rates ($\theta = 0, 1$ or 0.5), for given removal rates $\alpha = 1$ and 0.5 .

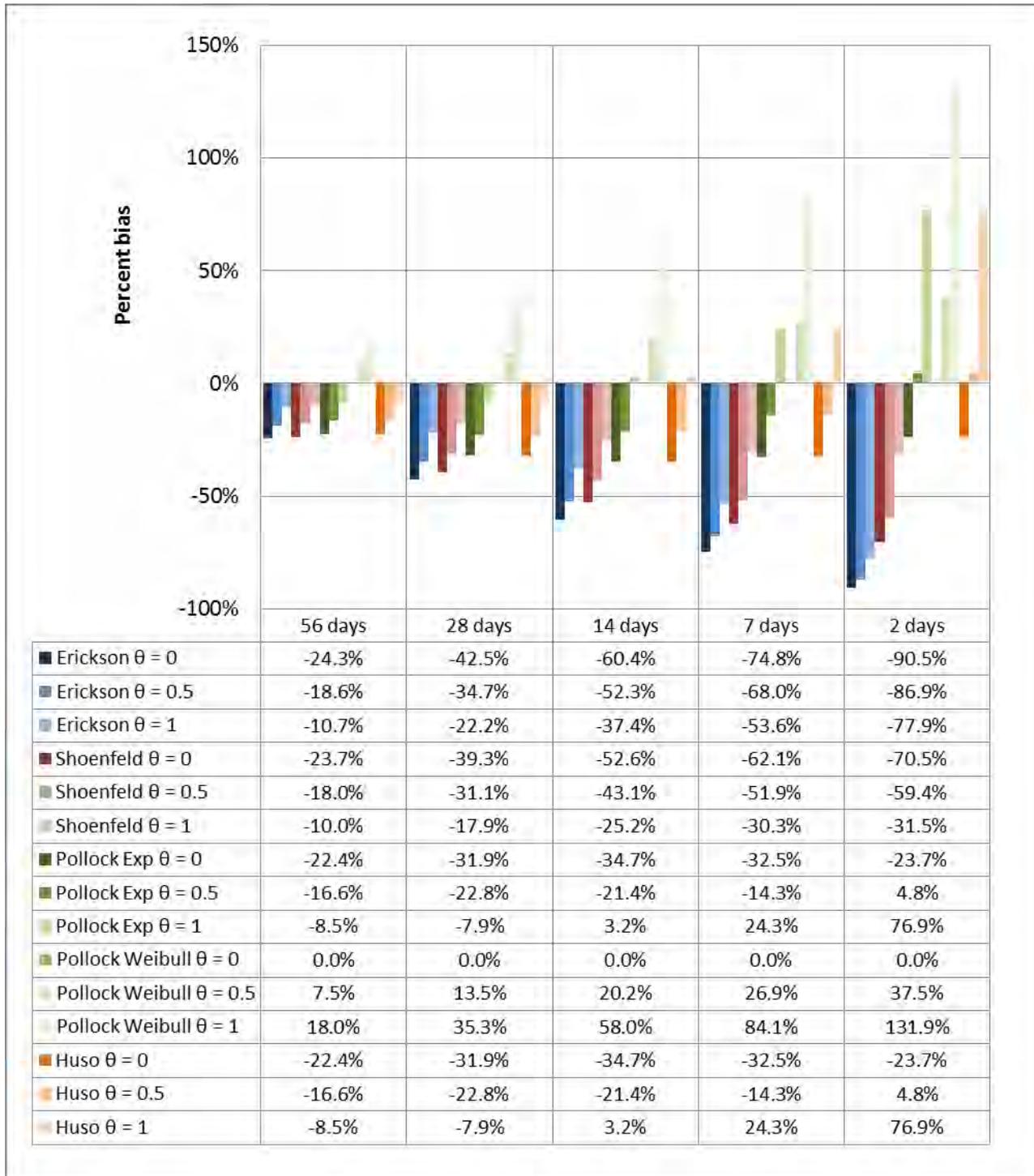
Figure 20: Comparison of Bias in Estimators at Various Search Intervals and “Bleed Through” (θ) Assumptions with Removal Rate $\alpha=1$



Where θ represents the percentage of carcasses neither discovered nor removed during one search interval that remain available to be discovered in later searches

Source: Dr. Robert Wolpert

Figure 21: Comparison of Bias in Estimators at Various Search Intervals and “Bleed Through” (θ) Assumptions with Removal Rate $\alpha = 0.5$



Where θ represents the percentage of carcasses neither discovered nor removed during one search interval and remain available to be discovered in later searches

Source: Dr. Robert Wolpert

The degree of bias among the equations is a function of many issues but, in all cases, it is a function of the inherent assumptions underlying the equation characteristics. Even when biased, if search intervals are *long* relative to mean persistence times, all four estimators give about the same answers. But if search intervals are *short* relative to mean persistence times, large differences among the equations are possible. In fact, it is very possible that, with short intervals, the results of the equations could differ by a factor of 3 or 4. For example, Shoenfeld's and Huso's estimators will differ by a factor of 3 or 4 or so if the search proficiency is 25 percent or 33 percent or so, because Huso assumes zero percent bleed-through and Shoenfeld assumes 100 percent bleed-through.

Conclusions and Recommendations

CalWEA's study provides new insights that could enhance the existing methods and procedures found in the Guidelines and other pre- and post-construction fatality monitoring guidelines used in the United States and internationally. Four major implications of this work and the corresponding recommendations are outlined here.

- (1) Traditional fatality estimators do not account for time-dependence of carcass persistence and searcher proficiency, or for "bleed-through."

Recommendation: Use the proposed new Partial Periodic Estimator and integrated detection probability trial method (proposed in Appendices A and B, respectively).

- (2) Traditional estimators can have high degrees of bias depending on the search interval, mean persistence, and bleed-through rate (θ) of the field data collected.

Recommendation: Do not use traditional estimators in conditions that produce levels of bias that are unacceptable for the intended purpose. Caution is particularly warranted where short search intervals have been used.

- (3) Use of traditional estimators has resulted in an unknown degree of bias in the literature.

Recommendation: Carefully consider the value of metrics like "industry average" before applying them in policy or project-specific decisions.

- (4) Previously generated fatality estimates used for project evaluation or broader purposes could be recalculated using the proposed new Partial Periodic Estimator, provided the key input variables (search interval, mean persistence, etc.) can be collected from the original studies and reasonable assumptions made about searcher proficiency probability distributions and θ values.

Recommendation: Going forward, use a standardized approach to generate unbiased project-specific results that may be compared with each other, and to generate meaningful and unbiased industry averages and totals.

These implications and recommendations are briefly discussed here.

Current estimators do not account for time-dependent processes and “bleed-through.” Detection probability is now known to be sensitive to time-dependent processes of carcass persistence, searcher proficiency, and bleed-through (θ), and that the traditional fatality estimators do not account for these influences. Therefore a new Partial Periodic Estimator (Appendix A) and an integrated detection probability field-trial methodology (Appendix B) are proposed and recommended that incorporate:

- Trials for searcher proficiency & carcass removal rates conducted simultaneously (vs. independent trials)

Further, the Guidelines on these issues are recommended to be revisited.

Care must be taken to avoid unacceptable bias when using current fatality estimators. The four traditional fatality estimators reviewed (Pollock, Erickson & Johnson, Shoenfeld and Huso) are now shown to have high degrees of bias depending on the search interval, mean persistence, and the proportion of bleed-through (θ) occurring in the field. Therefore these estimators are not recommended for use in conditions that produce unacceptable levels of bias (see Figures 20-21) unless biases can be corrected.

Note that “unacceptable” bias depends on circumstance and degree of accuracy needed.

- The inaccuracy of an estimate for a specific project may or may not be of consequence.
- The importance of accuracy or just precision depends on the sensitivity of the species, regulatory requirements, etc.

While individual project results are likely to be inaccurate, precise comparisons internal to a given project may still be useful provided the project studies are consistent with each other.

Use of previous study estimates

Previously generated study estimates can be used with some confidence in decision making where a persistence trial has produced a reliable mean value, providing that mean persistence time is shorter than the search interval (noting also that, in some cases, mean persistence will also have to be recalculated because of some common errors in methods of calculating this mean). If the persistence time is longer than the search interval, the estimate will be unreliable. If the mean is comparable to the search interval, the estimate will vary in the range of 30-40 percent.

Caution should be taken with metrics such as “Industry Average”

The findings in this project highlight the degree of difficulty that occurs when comparing mortality estimates among individual studies, particularly when the individual studies are not conducted with a standardized survey design. A number of factors make between-study results difficult, and also negate the ability to compare the results from a single study to an industry-wide average. For example, the following elements can negate the ability to compare mortality results on a national or state-wide basis: (1) differing approaches to treating the resulting survey data (*e.g.*, compiling data across individual turbines), (2) differing approaches to calculating inputs to the estimating equations (*e.g.*, estimation of mean persistence time), (3) the use of different equations, and (4) inconsistent survey design and field methods. Any industry average, therefore, will reflect a large variation among sites not due to variation in mortality, but due to the specific methods used to generate the mortality values. Therefore, a standardization of methods used to evaluate wind facility impacts is recommended, based in part on the findings of this report.

Considerable caution is in order when comparing individual project estimates to industry averages, given the possible level of bias in, and lack of comparability among, each of the source studies that are used to calculate the industry average. Similar cautions are in order when considering national total mortality figures.

Future Research

The results and findings of this study provide insight into needed changes in current monitoring practices and fatality estimation procedures at wind facilities. The existing estimating equations could be enhanced and improved with the addition of time-dependent processes for searcher proficiency and carcass persistence that are a function of environmental conditions. Appendix A presents a proposed new equation that incorporates these terms, and Appendix B outlines the key components for detection probability trial survey methods to support the proposed new estimator. Field testing the new estimating equation and protocols was beyond the scope of this study and report.

The Altamont study site provided a unique venue for studying fatality under changing conditions, and while all of the findings of this study will not directly translate to other sites, the general principles and findings should be applicable. The major findings of this study should hold generally for all wind facilities. However, the degree to which the vegetation height, time-based searcher efficiency, and other factors that were found influential in this study are transferable to other locations and conditions is explicitly unknown. Therefore, additional studies may provide insights on fatality estimation as a function of topographical, climatological, and environmental conditions.

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APPENDIX A: A New Equation for Estimating Avian Mortality Rates

A Partially Periodic Equation for Estimating Avian Mortality Rates

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Abstract

A key issue in assessing the environmental impact of wind plants for electrical power generation is the rate of mortality of birds and bats caused by collisions with turbines. The direct approach of counting and removing carcasses near a turbine facility at designated time intervals must be corrected in some way for the effects of removal by scavengers, detection failure, and other systematic biases. A number of authors have proposed estimation formulas intended to adjust for these, based on somewhat different assumptions about the underlying processes.

One significant issue on which these authors differ is whether or not bird carcasses present but *not* discovered and removed in the search ending one time interval, might possibly be discovered and counted in the search ending a later interval. The “periodic” estimator proposed by Shoenfeld (2004), for example, is based on a model in which any undiscovered carcasses may be found in later searches, while the aperiodic estimators proposed by Pollock (CEC, 2007, Appendix F) and by Huso (2011) are both based on the assumption that each interval begins with no discoverable carcasses.

We present a unified “partially periodic” structure that encompasses all of these estimators, in which a specified fraction of undiscovered carcasses remain discoverable in future searches. It includes that of Shoenfeld and those of Pollock and of Huso as special cases in which that fraction is 100% or 0%, respectively. The proposed estimator also accommodates arbitrary removal time distributions, avoiding the unrealistic assumption of exponential removal distributions implicit in the estimation formulas of Shoenfeld and Huso.

1 Introduction

The data we consider will be repeated counts of bird or bat carcasses made in designated search areas near each of several wind turbines. Denote by C_{ij} the count of carcasses in the designated area near the j th wind turbine by a Search Team at the end of the i th time interval, of length I_{ij} days, and by s_{ij} the search proficiency (discovery probability for a

carcass present at the time of the search) and by t_{ij} the average length of time (in days) before a new carcass is removed by scavengers.

For the special case of exponentially-distributed removal times, the proposed estimator (derived in Section (2.1) and generalized to arbitrary removal distributions in Section (2.2)) is:

$$\hat{M}_{ij}^* = \frac{C_{ij}I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left\{ \frac{e^{I_{ij}/\hat{t}_{ij}} - \theta(1 - \hat{s}_{ij})}{e^{I_{ij}/\hat{t}_{ij}} - 1} \right\}, \quad (1)$$

where $\theta \in [0, 1]$ is the fraction of undiscovered carcasses that remain discoverable in future searches. This includes as special cases each four of the estimators compared in (Wolpert, 2012; Warren-Hicks *et al.*, 2012):

Shoenfeld: For $\theta = 1$, indicating that *all* unremoved carcasses are discoverable eventually, this is exactly Shoenfeld's (Shoenfeld, 2004) estimator

$$\hat{M}_{ij}^s = \frac{C_{ij}I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left\{ \frac{e^{I_{ij}/\hat{t}_{ij}} - 1 + \hat{s}_{ij}}{e^{I_{ij}/\hat{t}_{ij}} - 1} \right\}. \quad (2)$$

Thus Eqn (1) may be viewed as a generalization of Shoenfeld's estimator to *partial* periodicity, and that presented in Section (2.2) a further generalization to arbitrary removal distributions.

Pollock: Pollock's mortality estimator (CEC, 2007, Appendix F)

$$\hat{M}_{ij}^p = \frac{C_{ij}}{\hat{p}_{ij}\hat{s}_{ij}} \quad (3a)$$

depends explicitly on \hat{p}_{ij} , the estimated "average probability a carcass will remain until the next search". For exponentially-distributed removal times this is $\hat{p}_{ij} = [1 - e^{-I_{ij}/\hat{t}_{ij}}]\hat{t}_{ij}/I_{ij}$, so in this case \hat{M}_{ij}^p may be expressed as

$$\hat{M}_{ij}^{p:E} = \frac{C_{ij}I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left[1 - e^{-I_{ij}/\hat{t}_{ij}} \right]^{-1} \quad (\text{Exponential removal}), \quad (3b)$$

the special case $\theta = 0$ of Eqn (1).

Huso: Huso's estimator (Huso, 2011) is identical to Pollock's for exponentially-distributed removal times (unless $I_{ij} > 4.6 \hat{t}_{ij}$, in which case \hat{M}_{ij}^H is about 1% larger), so it too is the special case of Eqn (1) with $\theta = 0$:

$$\hat{M}_{ij}^H = \frac{C_{ij}I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left[1 - e^{-I_{ij}/\hat{t}_{ij}} \right]^{-1}. \quad (4)$$

Erickson: Erickson & Johnson’s estimator (Erickson *et al.*, 1998; Johnson *et al.*, 2003)

$$\hat{M}_{ij}^{\text{EJ}} = \frac{C_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \quad (5)$$

would be (1) in the special case of $\theta = (1 - \hat{s}_{ij})^{-1}$. Note $\theta > 1$ here, because this estimator’s assumption that the system is in steady-state at each search is inconsistent with the usual practice of removing carcasses upon discovery by Search Teams.

Thus each of these estimators corresponds to some specific choices for removal distribution and for the parameter θ — but maybe not the choices one would prefer on further reflection.

Some have expressed the opinion that both 100% persistence of undiscovered carcasses (*i.e.*, $\theta = 1$, as in Shoenfeld’s estimator) and 100% disappearance of them (*i.e.*, $\theta = 0$, as in Pollock’s and Huso’s) are unrealistic, and that the truth lies somewhere in between. Also, evidence (Bispo *et al.*, 2012; Warren-Hicks *et al.*, 2012, *p.* 21ff) suggests that other survival distributions fit removal times better than exponential distributions.

2 The Model Underlying the New Partially Periodic Estimator

Suppose (as usual) that carcasses arrive in a Poisson stream with average daily rate m_{ij} and, following Shoenfeld, that the process is “periodic” in the sense that consecutive interval lengths I_{ij} (in days), mean mortality rates m_{ij} (per day), and search proficiency probabilities s_{ij} vary slowly in time— and hence do not vary with index i (though they may still differ across turbines, indexed by j). In contrast to Shoenfeld, we now assume that while all unscavenged carcasses arriving during the current period are discoverable in the search ending that period, only a fraction $\theta_j \in [0, 1]$ of those unscavenged carcasses not discovered in that search will remain discoverable for future searches.

2.1 Exponential Removal Times

The average number of discoverable carcasses at the end of any period (call it \mathbf{g}_1) will be the sum of those carcasses (if any) remaining unscavenged and undiscovered by earlier searches, plus those carcasses arriving at times uniformly distributed over the present interval and remaining unremoved until its end. For the case of exponentially-distributed persistence with rate $r_{ij} > 0$, this is

$$\mathbf{g}_1 = e^{-r_{ij} I_{ij}} \mathbf{g}_0 + m_{ij} \int_0^{I_{ij}} e^{-r_{ij}(I_{ij}-s)} ds$$

where \mathbf{g}_0 denotes the average number of discoverable carcasses at the beginning of the period. By periodicity this is $\mathbf{g}_0 = \mathbf{g}_1 \theta_j (1 - s_{ij})$, so

$$\mathbf{g}_1 = e^{-r_{ij} I_{ij}} \mathbf{g}_1 \theta_j (1 - s_{ij}) + \frac{m_{ij}}{r_{ij}} [1 - e^{-r_{ij} I_{ij}}].$$

Collecting terms and solving, and using $r_{ij} = 1/t_{ij}$ and $\mathbb{E}M_{ij} = m_{ij} I_{ij}$,

$$\mathbf{g}_1 = \frac{\mathbb{E}M_{ij}t_{ij}}{I_{ij}} \left\{ \frac{1 - e^{-I_{ij}/t_{ij}}}{1 - \theta_j(1 - s_{ij})e^{-I_{ij}/t_{ij}}} \right\}.$$

For searchers with proficiency s_{ij} the expected carcass count is $\mathbb{E}[C] = s_{ij}\mathbf{g}_1$, leading to the estimator

$$\hat{M}_{ij}^* = \frac{C_{ij}I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left\{ \frac{1 - \theta_j(1 - \hat{s}_{ij})e^{-\hat{z}_{ij}}}{1 - e^{-I_{ij}/t_{ij}}} \right\}. \quad (1)$$

2.2 The General Case

The exponential distribution, commonly used for modeling removal times, features constant *hazard*—the probability of removal by scavengers in a short time interval is approximately the same constant multiple $h \Delta$ of the interval’s length Δ , irrespective of the age t of the carcass at the start of the interval. Evidence suggests that in fact the hazard rate $h(t)$ varies with carcass age, and that (at least for large t) it decreases. Warren-Hicks *et al.* (2012) found that Weibull distributions with shape parameter $\alpha < 1$ (whose hazard $h(t)$ decreases at rate $t^{-(1-\alpha)}$) fit the observed removal times far better than exponential distributions, while Bispo *et al.* (2012) found that log normal and log logistic distributions (whose hazards decrease at rate $1/t$ for large t) or Weibull distributions fit their data better than the exponential. In this section we develop a partially-periodic estimator for arbitrary removal distributions.

Denote by $\bar{F}_j(t) = \mathbb{P}[\tau > t]$ the complimentary CDF, or survival function, for removal times $\tau \geq 0$, and by

$$Q_j(t) \equiv \mathbb{E}[\tau \wedge t] = \int_0^t \bar{F}_j(x) dx \quad (6)$$

the *integrated survival function* or “ISF”. Clearly $Q_j(t) \leq t$ (because $\bar{F}_j(x) \leq 1$), and $Q_j(t)$ increases as $t \rightarrow \infty$ to the mean removal time $t_{ij} \equiv \mathbb{E}[\tau] = \int_0^\infty \bar{F}_j(x) dx$ (which may be infinite for some distributions). As before denote by $\theta_j \in [0, 1]$ the average fraction of undiscovered carcasses that remain discoverable in later searches, by s_{ij} the probability a carcass present at the time of a search will be discovered, and by m_{ij} the average daily rate of mortality.

The discoverable carcasses on the ground at any given time include “new” carcasses that arrived since the last search along with those “old” ones that arrived in earlier periods and eluded discovery by search teams. Thus if $q_j \equiv \theta_j(1 - s_{ij})$ denotes the probability that a carcass is undetected in a search and remains discoverable for future searches, then t days after the most recent search the expected number $g_j(t)$ of discoverable carcasses is the sum

$$g_j(t) = \int_0^t m_{ij} \bar{F}_j(t-s) ds + \sum_{k \geq 1} q_j^k \int_{-kI_{ij}}^{-(k-1)I_{ij}} m_{ij} \bar{F}_j(t-s) ds$$

of those arriving in the current period and those arriving earlier who went undetected in some number $k \geq 1$ of searches. Changing variables to $x = t - s$,

$$\begin{aligned} &= m_{ij} \int_0^t \bar{F}_j(x) dx + m_{ij} \sum_{k \geq 1} q_j^k \int_{t+(k-1)I_{ij}}^{t+kI_{ij}} \bar{F}_j(x) dx, \\ &= m_{ij} Q_j(t) + m_{ij} \sum_{k \geq 1} q_j^k [Q_j(t + kI_{ij}) - Q_j(t + (k-1)I_{ij})], \end{aligned}$$

(recall $Q_j(t) := \int_0^t \bar{F}_j(x) dx$), a telescoping series with sum

$$g_j(t) = m_{ij}(1 - q_j) \sum_{n \geq 0} q_j^n Q_j(t + nI_{ij}). \quad (7)$$

The expected number of carcasses counted $\mathbf{E}[C_{ij}]$ will be a fraction s_{ij} of $g_j(I_{ij})$, and the average daily mortality rate is $m_{ij} = \mathbf{E}[M_{ij}]/I_{ij}$, leading to the new Partially Periodic estimator

$$\hat{M}_{ij}^* = \frac{C_{ij} I_{ij}}{R_{ij} \hat{s}_{ij}} \quad (8)$$

where $R_{ij} = g_j(I_{ij})/m_{ij}$ is given by

$$R_{ij} \equiv [1 - \theta_j(1 - s_{ij})] \sum_{k \geq 1} [\theta_j(1 - s_{ij})]^{k-1} Q_j(k I_{ij}). \quad (9a)$$

This sum always converges to a well-defined and finite quantity $R_{ij} \leq I_{ij}/[1 - \theta_j(1 - s_{ij})]$ for any removal distribution, even if $t_{ij} = \mathbf{E}[\tau]$ is infinite.

For $\theta_j = 0$ only the first term ($k = 1$) contributes to the sum in (9a). In that case $R_{ij} = Q_j(I_{ij}) = I_{ij} p_{ij}$ where $p_{ij} \equiv \int_0^{I_{ij}} \bar{F}_j(I_{ij} - t) dt / I_{ij}$ is the average probability a carcass is unremoved until the next search, and (8) reduces to Pollock's estimator $\hat{M}_{ij}^P = C_{ij} / \hat{p}_{ij} \hat{s}_{ij}$ for any underlying removal distribution.

For $\theta_j > 0$, the k th term in the sum of (9a) accounts for carcasses that went undiscovered through $(k-1)$ searches and unremoved for about $k \times I_{ij}$ days. It's not surprising that only a few terms are necessary (usually just two or three) to evaluate R_{ij} with sufficient accuracy. Bounds are presented in Section (3) for finite approximations including simple truncation,

$$R_{ij} \approx R_{ij}^n \equiv [1 - \theta_j(1 - s_{ij})] \sum_{k=1}^n [\theta_j(1 - s_{ij})]^{k-1} Q_j(k I_{ij}). \quad (9b)$$

2.3 Parametric Examples

In this section we consider several parametric distributions commonly used for survival or lifetime analysis. To facilitate introducing a regression setting later to reflect dependence on covariates such as vegetation height, in each case we parametrize the distribution with a *rate* parameter $r_{ij} > 0$ and perhaps one or more shape parameters.

2.3.1 Exponential Persistence

For the exponential distribution, the survival function, mean, and ISF are

$$\bar{F}_j(t) \equiv \mathbb{P}[\tau > t] = e^{-r_{ij} t} \quad (10a)$$

$$t_{ij} \equiv \mathbb{E}[\tau] = \int_0^\infty e^{-r_{ij} x} dx = 1/r_{ij} \quad (10b)$$

$$Q_j(t) \equiv \mathbb{E}[\tau \wedge t] = \int_0^t e^{-r_{ij} x} dx = t_{ij}[1 - e^{-t/t_{ij}}] \quad (10c)$$

for $t \geq 0$ so (7) and (9a) are geometric series with sums

$$g_j(t) = m_{ij} t_{ij} \left\{ 1 - \frac{1 - q_j}{1 - q_j e^{-I_{ij}/t_{ij}}} e^{-t/t_{ij}} \right\} \quad (10d)$$

$$R_{ij} = t_{ij} \frac{e^{I_{ij}/t_{ij}} - 1}{e^{I_{ij}/t_{ij}} - q_j} \quad (10e)$$

with $q_j \equiv \theta_j(1 - s_{ij})$. This leads to the exponential removal Partially Periodic estimator

$$\hat{M}_{ij}^* \equiv \frac{C_{ij} I_{ij}}{\hat{R}_{ij} \hat{s}_{ij}} = \frac{C_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left\{ \frac{e^{I_{ij}/\hat{t}_{ij}} - \theta_j(1 - s_{ij})}{e^{I_{ij}/\hat{t}_{ij}} - 1} \right\}$$

identical to (1). It reduces further to $\hat{M}_{ij}^{P:E}$ of Eqn (3b) or \hat{M}_{ij}^H of Eqn (4) for $\theta_j = 0$ and to \hat{M}_{ij}^S of Eqn (2) for $\theta_j = 1$.

2.3.2 Weibull Persistence

The survival function, mean, and ISF for the Weibull distribution with shape $\alpha > 0$ are

$$\bar{F}_j(t) \equiv \mathbb{P}[\tau > t] = e^{-(r_{ij} t)^\alpha} \quad (11a)$$

$$t_{ij} \equiv \mathbb{E}[\tau] = \int_0^\infty e^{-(r_{ij} x)^\alpha} dx = \Gamma(1 + \frac{1}{\alpha})/r_{ij}, \quad (11b)$$

$$Q_j(t) \equiv \mathbb{E}[\tau \wedge t] = \int_0^t e^{-(r_{ij} x)^\alpha} dx = P(\frac{1}{\alpha}, (r_{ij} t)^\alpha) t_{ij} \quad (11c)$$

where $\Gamma(a)$ and $P(a, x)$ denote the gamma and incomplete gamma functions, respectively (Abramowitz and Stegun, 1964, §6.5), so $\hat{M}_{ij}^* \equiv C_{ij} I_{ij} / \hat{R}_{ij} \hat{s}_{ij}$ with

$$g_j(t) = t_{ij}(1 - q_j) m_{ij} \sum_{k \geq 0} q_j^k P(\frac{1}{\alpha}, [r_{ij}(t + k I_{ij})]^\alpha) \quad (11d)$$

$$R_{ij} \equiv t_{ij}(1 - q_j) \sum_{k \geq 1} q_j^{k-1} P(\frac{1}{\alpha}, (k r_{ij} I_{ij})^\alpha) \quad (11e)$$

again with $q_j \equiv \theta_j(1 - s_{ij})$ (see Section (3) for finite approximations). This is illustrated in Figure (1) and Section (2.5).

2.3.3 Log Normal Persistence

The survival function, mean, and ISF for the log normal distribution are

$$\bar{F}_j(t) \equiv \mathbb{P}[\tau > t] = \Phi(-\alpha \log_j(r_{ij} t)) \quad (12a)$$

$$t_{ij} \equiv \mathbb{E}[\tau] = \int_0^\infty \bar{F}_j(x) dx = e^{1/2\alpha^2}/r_{ij} \quad (12b)$$

$$Q_j(t) \equiv \mathbb{E}[\tau \wedge t] = \int_0^t \bar{F}_j(x) dx = \Phi(-\alpha \log_j(r_{ij} t)) t + \Phi(\alpha \log_j(r_{ij} t) - \frac{1}{\alpha}) t_{ij} \quad (12c)$$

where $\Phi(z)$ is the standard Normal CDF, and so $\hat{M}_{ij}^* \equiv C_{ij} I_{ij} / \hat{R}_{ij} \hat{s}_{ij}$ with

$$g_j(t) = (1 - q_j) m_{ij} \sum_{k \geq 0} q_j^k Q_j(t + k I_{ij}) \quad (12d)$$

$$R_{ij} \equiv (1 - q_j) \sum_{k \geq 1} q_j^{k-1} Q_j(k I_{ij}). \quad (12e)$$

The log normal distribution is more commonly parametrized by the mean $\mu = -\log r_{ij}$ and variance $\sigma^2 = 1/\alpha^2$ of $\log \tau$.

2.3.4 Log Logistic Persistence

The survival function, mean, and ISF for the log logistic (or ‘‘Fisk’’) distribution are

$$\bar{F}_j(t) \equiv \mathbb{P}[\tau > t] = [1 + (r_{ij} t)^\alpha]^{-1} \quad (13a)$$

$$t_{ij} \equiv \mathbb{E}[\tau] = \int_0^\infty \frac{dx}{1 + (r_{ij} x)^\alpha} = \begin{cases} \frac{\pi/\alpha}{r_{ij} \sin(\pi/\alpha)} & \alpha > 1 \\ \infty & \alpha \leq 1 \end{cases} \quad (13b)$$

$$Q_j(t) \equiv \mathbb{E}[\tau \wedge t] = \int_0^t \frac{dx}{1 + (r_{ij} x)^\alpha} = t {}_2F_1\left(1, \frac{1}{\alpha}; 1 + \frac{1}{\alpha}; -(r_{ij} t)^\alpha\right) \quad (13c)$$

$$= \frac{t}{1 + (r_{ij} t)^\alpha} {}_2F_1\left(1, 1; 1 + \frac{1}{\alpha}; \frac{1}{1 + (r_{ij} t)^{-\alpha}}\right)$$

where ${}_2F_1(a, b; c; z)$ is Gauss’ hypergeometric function (Abramowitz and Stegun, 1964, §15.1). Note $t_{ij} = \infty$ is possible for this distribution. Again $\hat{M}_{ij}^* \equiv C_{ij} I_{ij} / \hat{R}_{ij} \hat{s}_{ij}$ with

$$g_j(t) = (1 - q_j) m_{ij} \sum_{k \geq 0} q_j^k Q_j(t + k I_{ij}) \quad (13d)$$

$$R_{ij} \equiv (1 - q_j) \sum_{k \geq 1} q_j^{k-1} Q_j(k I_{ij}) \quad (13e)$$

with finite approximations given in Eqn (19). The log logistic distribution is more commonly parametrized by the median $m = -\log r_{ij}$ and scale $s = 1/\alpha$ of $\log \tau$.

2.4 Estimating Parameters and Persistence

The parameters governing removal distributions are usually estimated with the help of *removal trials* in which some number C_0 of carcasses are placed at known locations at time $t_0 = 0$, then checked on each of some number n of succeeding days $0 < t_1 < t_2 < \dots < t_n$ revealing counts $C_0 \geq C_1 \geq C_2 \geq \dots \geq C_n$ of remaining carcasses (see, for example, Erickson *et al.*, 2008, §2.6). To simplify some formulas below, set $t_{n+1} \equiv \infty$ and $C_{n+1} \equiv 0$, with $\bar{F}_j(\infty) = 0$.

2.4.1 Estimating Parameters

If we were able to observe the exact lengths of time $\{\tau_k\}$ until each carcass's removal, the log likelihood function for the rate parameter r_{ij} and shape parameter α of any of the distributions considered in Sections 2.3.2–2.3.4 would be

$$\ell(\alpha, r_{ij}) = \sum_k \log f_j(\tau_k; \alpha, r_{ij})$$

where $f_j(t; \alpha, r_{ij}) \equiv -(\partial/\partial t)\bar{F}_j(t; \alpha, r_{ij})$ denotes the probability density function (pdf) for removal, with the parameter dependence made explicit. Our data are censored, however, to only the counts C_m of $\{\tau_k\}$ in the intervals (t_m, ∞) for $0 \leq m \leq n$, leading to the multinomial log likelihood

$$\ell(\alpha, r_{ij}) = \sum_{m=0}^n [C_m - C_{m+1}] \log (\bar{F}_j(t_m; \alpha, r_{ij}) - \bar{F}_j(t_{m+1}; \alpha, r_{ij})) \quad (14)$$

from which estimates $\hat{\alpha}$, \hat{r}_{ij} can be found numerically. For equally-spaced search intervals $t_m = m\Delta$ and exponentially-distributed removal, a closed-form expression for the rate maximum likelihood estimators (MLEs) of r_{ij} and of the mean persistence time $t_{ij} = 1/r_{ij}$ are available:

$$\hat{r}_{ij} = \frac{1}{\Delta} \log \left\{ 1 + \frac{C_0 - C_n}{\sum_{1 \leq m \leq n} C_m} \right\} = \frac{1}{\Delta} \log \left\{ \frac{\sum_{0 \leq m < n} C_m}{\sum_{0 < m \leq n} C_m} \right\}, \quad \hat{t}_{ij} = 1/\hat{r}_{ij} \quad (15)$$

Note this is quite different from the formula for estimating mean persistence suggested in (Erickson *et al.*, 2008, §3.3), which will systematically underestimate \hat{t}_{ij} . Searcher proficiency s_{ij} can also be estimated empirically from removal trial experimental data (see Section (2.4.3)).

2.4.2 Empirical Persistence

An alternative to the parametric models presented in Section (2.3) is to estimate R_{ij} directly from experimental data gathered in a removal trial. If θ_j and s_{ij} (and hence q_j) are known, then the maximum likelihood estimator (MLE) \hat{R}_{ij} can be computed from the MLE $\hat{Q}_j(t)$ of the ISF $Q_j(t)$.

Unfortunately the MLE for the ISF is not uniquely determined— the likelihood takes the same maximum value at all non-decreasing functions $Q(t)$ satisfying $\hat{Q}_j^-(t) \leq Q(t) \leq \hat{Q}_j^+(t)$, with lower and upper limits given by:

$$\hat{Q}_j^-(t) = \sum_{k=1}^n \frac{C_{k-1} - C_k}{C_0} (t_{k-1} \wedge t) + \frac{C_n}{C_0} t_n \quad \hat{Q}_j^+(t) = \sum_{k=1}^n \frac{C_{k-1} - C_k}{C_0} (t_k \wedge t) + \frac{C_n}{C_0} t \quad (16)$$

These lead in turn to lower and upper MLEs for R_{ij} of

$$\hat{R}_{ij}^\pm \equiv (1 - q_j) \sum_{k \geq 1} q_j^{k-1} \hat{Q}_j^\pm(k I_{ij})$$

and hence to Partially Periodic estimators

$$\hat{M}_{ij}^{*\pm} \equiv \frac{C_{ij} I_{ij}}{\hat{R}_{ij}^\mp \hat{s}_{ij}}. \quad (17)$$

The estimator \hat{M}_{ij}^{*+} generated from the lower MLE \hat{R}_{ij}^- is conservative, in that it will slightly overestimate M_{ij} on average, while on average that \hat{M}_{ij}^{*-} generated from \hat{R}_{ij}^+ will slightly underestimate M_{ij} . Note too that the sums for evaluating both R_{ij}^- and R_{ij}^+ entail at most $\lceil t_n/I_{ij} \rceil$ terms, since for $t > t_n$ the function $\hat{Q}_j^-(t)$ is constant and $\hat{Q}_j^+(t)$ is linear, leading to summable geometric series. The difference $[\hat{Q}_j^+(t) - \hat{Q}_j^-(t)]$ for $t \leq t_n$ are weighted averages of the inter-search intervals $[t_k - t_{k-1}]$, hence smaller than the largest such interval.

2.4.3 Regression

Now suppose that in a trial we have a vector of $p \geq 1$ covariates for each carcass that might affect the rate of removal, such as vegetation height or slope. Model the rate parameter r_{ij} for the k th carcass as

$$r_{ij} = \exp(X_k \cdot \beta)$$

for a p -dimensional vector β of regression coefficients, and denote by t_k^- the last search time $\{t_m\}$ the carcass was still present, and by t_k^+ the first search time the carcass was absent (or ∞ if the trial ended before it was removed). The log likelihood function of (14) becomes

$$\ell(\alpha, \beta) = \sum_k \log [\bar{F}_j(t_k^-; \alpha, \exp(X_k \cdot \beta)) - \bar{F}_j(t_k^+; \alpha, \exp(X_k \cdot \beta))] \quad (18)$$

which can be maximized numerically in the parameters α and β .

These covariates may also affect the probability of discovery s_{ij} , which could also be modeled in log-linear fashion as

$$s_{ij} = \exp(X_k \cdot \gamma)$$

for an uncertain p -dimensional vector γ of regression coefficients.

2.5 Illustration

Figure 1 shows a simulation of four $I_{ij} = 10$ d periods with average daily mortality rate of $m_{ij} = 3 \text{ d}^{-1}$. Persistence distributions are Weibull with shape $\alpha = 0.70$ and mean $t_{ij} = 15$ d; search proficiency is $s_{ij} = 30\%$, and $\theta_j = 50\%$ of carcasses remain discoverable in subsequent periods (a compromise between the Shoefeld and Pollock values). Counts of discoverable carcasses are shown as stair-step curve beginning and ending at open and filled squares, that increases (resp. decreases) by one with each new fatality (resp. removal by scavengers), and decreases by the number C_{ij} of carcasses discovered and removed by searchers (shown as a red downward arrow) and by an additional number that become undiscoverable. Expected numbers of discoverable carcasses are shown as smooth curves beginning and ending at open and filled circles. For these values, Erickson & Johnson’s estimator has a bias of -50% (*i.e.*, on average $\hat{M}_{ij}^{\text{EJ}} \approx \frac{1}{2}M_{ij}$) and Shoefeld’s \hat{M}_{ij}^{S} has bias -34.2% , because each assumes that *all* carcasses remain discoverable while in fact only half do. Pollock’s estimator $\hat{M}_{ij}^{\text{P:W}}$ has positive bias $+22.8\%$, because it assumes that *no* carcasses remain discoverable while in fact half do. Pollock’s $\hat{M}_{ij}^{\text{P:E}}$ (and Huso’s identical \hat{M}_{ij}^{H}) comes closer, with just $+2.7\%$ bias, because the reduction from its incorrect assumption of exponential removal distributions and the inflation from its incorrect assumption that intervals begin without discoverable carcasses nearly cancel out (a coincidence arising from our choice of parameter values). The new Partially Periodic estimator \hat{M}_{ij}^* has zero bias.

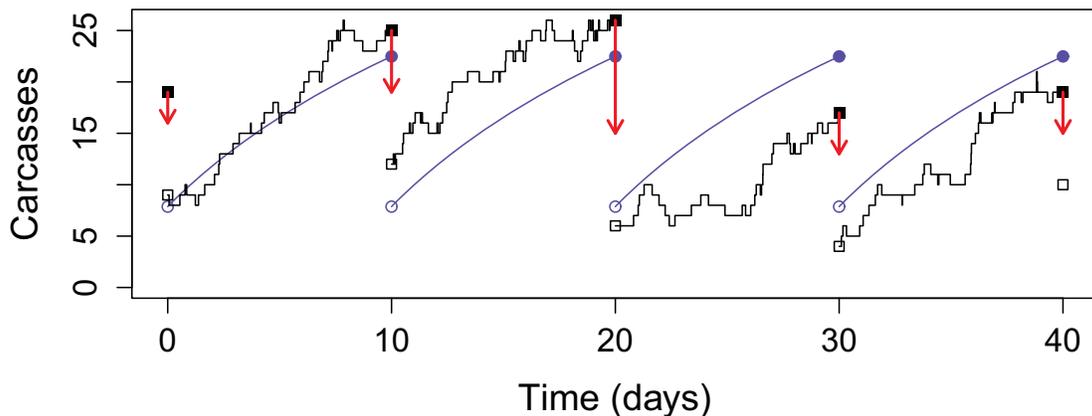


Figure 1: One draw (stair-step) from random distribution of all carcasses present at times t , for $0 \leq t \leq 40$ d. Expected value $g(t)$ is shown as solid line, beginning and ending search periods at open and filled circles, removal by search teams as downward arrows.

For exponential removal times ($\alpha = 1$), the new estimator will always lie between the estimators

$$\hat{M}_{ij}^{\text{S}} < \hat{M}_{ij}^* < \hat{M}_{ij}^{\text{P}}$$

of Shoefeld and Pollock (or, equivalently, Huso), with negligible differences among them whenever I_{ij}/\hat{t}_{ij} is large. The differences are larger with more frequent searches, but never exceed a factor of s_{ij} : for any I_{ij} and \hat{t}_{ij} , always $s_{ij}\hat{M}_{ij}^{\text{P}} \leq \hat{M}_{ij}^{\text{S}} \leq \hat{M}_{ij}^{\text{P}} \leq \hat{M}_{ij}^{\text{S}}/s_{ij}$.

3 Computation

3.1 Evaluating R_{ij}

Since $Q_j(t) \leq t$ and $Q_j(t) \leq \hat{t}_{ij}$, the truncation error incurred by approximating R_{ij} with just the first n terms as in Eqn (9b)

$$R_{ij}^n \equiv (1 - q_j) \sum_{k=1}^n q_j^{k-1} Q_j(k I_{ij})$$

is bounded above by both of

$$\begin{aligned} R_{ij} - R_{ij}^n &\leq (1 - q_j) \sum_{k>n} q_j^{k-1} k I_{ij} &&= q_j^n I_{ij} [n + 1/(1 - q_j)] \\ R_{ij} - R_{ij}^n &\leq (1 - q_j) \sum_{k>n} q_j^{k-1} \hat{t}_{ij} &&= q_j^n \hat{t}_{ij} \end{aligned}$$

and, by monotonicity, is bounded below by

$$R_{ij} - R_{ij}^n \geq (1 - q_j) \sum_{k>n} q_j^{k-1} Q_j(n I_{ij}) = q_j^n Q_j(n I_{ij}),$$

leading to the two-sided bound

$$R_{ij}^{n-} \equiv R_{ij}^n + q_j^n Q_j(n I_{ij}) \leq R_{ij} \leq R_{ij}^{n+} \equiv R_{ij}^n + q_j^n \min\left(\hat{t}_{ij}, I_{ij} [n + 1/(1 - q_j)]\right). \quad (19)$$

For the parameter values used in the simulation of Section (2.5) presented in Figure 1 the error is bounded by $(0.5 * (1 - 0.3)e^{-2.5})^N = (0.0288)^N$, so just $N = 2$ terms suffice for 99.92% accuracy.

3.2 Special Functions

The partially-periodic estimator \hat{M}_{ij}^* of Eqn (1) is simple to evaluate in closed form under the assumption that removal times have exponential distributions, as is the estimator \hat{M}_{ij}^{*+} of Eqn (17) based on non-parametric empirical estimation of removal distributions. The version of \hat{M}_{ij}^* for log normal removal distributions presented in Section (2.3.3) requires only the cumulative normal distribution function $\Phi(z)$, available in virtually every computing environment, but the estimators for assumed log logistic or Weibull removal distributions require slightly less commonly used functions.

The complete and incomplete gamma functions $\Gamma(a)$ and $P(a, x)$ required for the Weibull estimator of Section (2.3.2) are included in R (R Core Team, 2012) as `gamma(a)` and `pgamma(x, a)`, respectively, and are also included in MATLAB, Mathematica, the gnu scientific library (gsl: Galassi *et al.*, 2009), and other standard computational environments. In Microsoft Excel

they are available as $\Gamma(a) = \text{EXP}(\text{GAMMALN.PRECISE}(a))$ and $P(a, x) = \text{GAMMA.DIST}(x, a, 1, \text{TRUE})$, respectively (for versions of MS Excel prior to 10, use `GAMMALN` and `GAMMADIST` instead).

Gauss' hypergeometric function ${}_2F_1(a, b; c; z)$ needed for the log logistic removal models of Section 2.3.4) is available in `MATLAB`, in `Mathematica`, and `R` (using the `gs1` package). While it's not included in MS Excel, certain special cases are— for example, for $\alpha = 2$, $Q_j(t) = \arctan(r_{ij}t)/r_{ij}$ and (13e) becomes

$$\begin{aligned} R_{ij} &= (1 - q_j) \sum_{k \geq 1} q_j^{k-1} \arctan(kr_{ij}I_{ij})/r_{ij} \\ &\approx R_{ij}^{n-} \equiv (1 - q_j) \sum_{k=1}^n q_j^{k-1} \arctan(kr_{ij}I_{ij})/r_{ij} + q_j^n \arctan(nr_{ij}I_{ij})/r_{ij} \end{aligned}$$

for any small n , easily evaluated in Excel, with an error bounded by Eqn (19).

4 Non-constant Removal Rates and Proficiencies

One part of a 2011 study by the California Wind Energy Association (Warren-Hicks *et al.*, 2012) consisted of a removal trial in which a number of bird and bat carcasses were placed in known locations and followed for 45 days. At intervals, searches would be undertaken and each discovered carcass would be recorded. Periodically the trial coordinators would check each carcass to see if it had yet been removed by scavengers.

4.1 Estimating Proficiency Time Dependence

A key finding of this study is that the proficiency s_{ij} does *not* appear to be constant, unrelated to the age of the carcass— rather, that older carcasses have a lower probability of discovery than fresher carcasses. This is illustrated in Figure (2), which shows the data for small birds: 38 successful discoveries of carcasses aged 1–28 days, shown as small circles \circ at height $y = 1$, and 185 undiscovered carcasses, aged 1–45 days, shown as small circles at height $y = 0$ (a small jitter is added to the locations of each circle to reveal multiplicity). The overall average proficiency is $\bar{s}_{ij} = 17\%$, but there is clear evidence that $\bar{s}_{ij}(t)$ diminishes with carcass age t . The figure also shows an empirical exponential moving-average estimate (with a 5-day window) as a dashed red line, and the best fit with a logistic regression model shown as a solid blue curve:

$$\hat{s}_{ij}(t) = (1 + e^{0.6441+0.0911t})^{-1}, \quad (20a)$$

starting at $\hat{s}_{ij}(0) = 45.75\%$, falling to the overall average $\hat{s}_{ij}(10.3) = 17\%$ after about 10 days and continuing to fall down to a negligible $\hat{s}_{ij}(45) = 0.86\%$. The two curves are in substantial agreement throughout the range, suggesting that the logistic regression model is a good fit. The best fit for a model with simple exponentially-decreasing proficiency

$$\hat{s}_{ij}(t) = e^{-1.017-0.0777t} \quad (20b)$$

(in green) is virtually indistinguishable; we'll return to this one in Section (4.3).

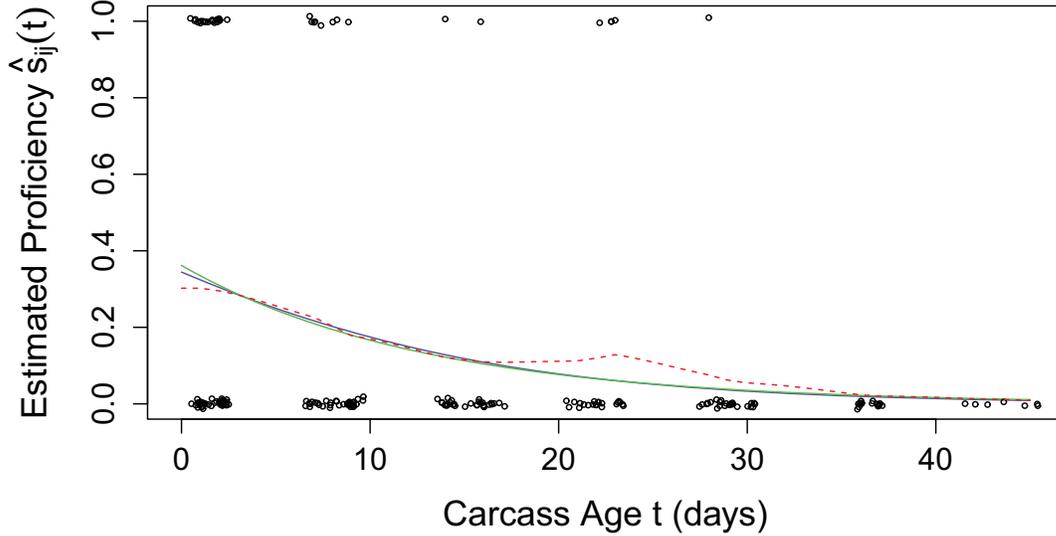


Figure 2: Time-dependence of search proficiency $\hat{s}_{ij}(t)$. Dashed red curve is nonparametric moving-average estimator (exponential window, width 5 days), solid blue and green curves are best logistic-regression and exponential fits (see Eqn (20)).

4.2 Estimating Removal Time Dependence

Other investigators (such as Bispo *et al.*, 2012) have reported earlier that exponential distributions offer a worse fit to empirical data on carcass removal by scavengers than several alternatives, and data from Warren-Hicks *et al.* (2012) reaffirm this finding. Figure (3) shows the upper and lower empirical survival curves (Kaplan and Meier, 1958) for these censored data, along with best fits for Exponential Distribution (dashed red curve) and Weibull (solid blue curve). Evidently the Weibull fits far better. Both log-logistic and log-normal fits are similar to Weibull, and far better than exponential, suggesting that hazard rate (daily probability of removal by scavengers), like search proficiency, diminishes over time. The best-fit Weibull had shape parameter $\hat{\alpha} = 0.4606$ with a Standard Error of 0.0532 d, about 10.31 Standard Errors below the value $\alpha = 1$ corresponding to the exponential distribution, leading to emphatic rejection of exponentially-distributed removal. The estimated rate parameter is $\hat{r}_{ij} = 0.07944$, for a mean persistence time of

$$\hat{t}_{ij} = \Gamma(1 + 1/\hat{\alpha})/\hat{r}_{ij} = 29.64 \text{ d},$$

substantially longer than the estimate $\hat{t}_{ij} = 16.68 \text{ d}$ under the exponential model which (see Figure (3)) systematically underestimates early removal and overestimates late removal.

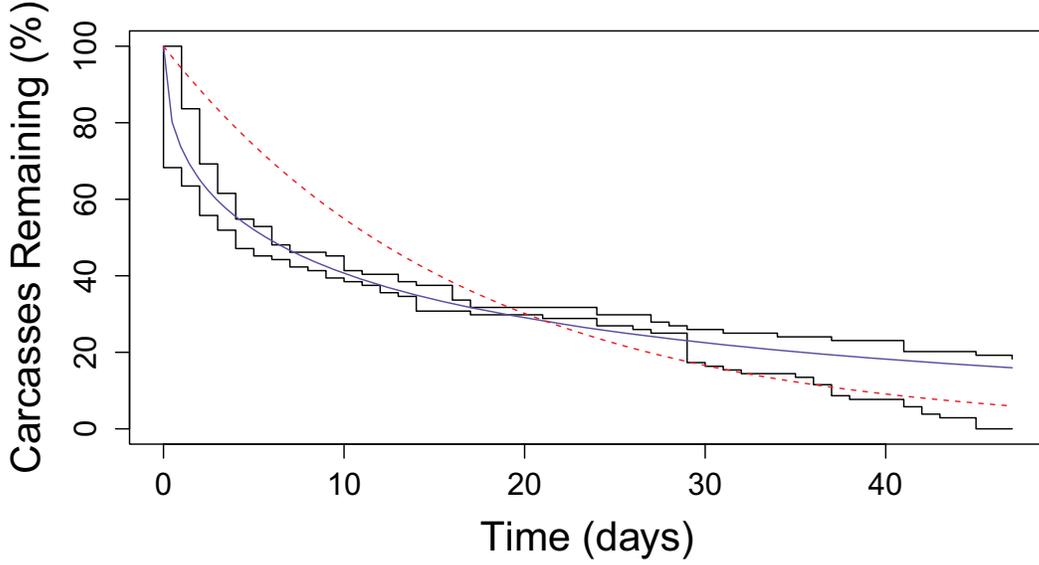


Figure 3: Upper and lower empirical (Kaplan-Meier) Removal curves (solid black stair-steps), with best Weibull fit (solid blue curve) and best Exponential fit (dashed red curve).

4.3 Adapting the Model and Mortality Estimators

Although the model and mortality estimator of Section (2.2) makes no assumptions on removal distributions, and the parametric examples presented in Section (2.3) include the Weibull, Lognormal, and Log Logistic, all of which fit our data well, each of these (along with all other published mortality estimators we are aware of) are built on the assumption that search proficiency is constant, unrelated to carcass age; in Section (4.1) we showed that this assumption appears to be false. In this section we will adapt the model and estimators of Sections (2.2) and (2.3) to accommodate age-dependent proficiency.

By the same argument and changes-of-variables that led to Eqn (9a), the number of carcasses counted at each search will have a Poisson distribution with mean

$$c_{ij} = m_{ij} \sum_{k=0}^{\infty} \theta^k \int_{kI_{ij}}^{(k+1)I_{ij}} \bar{F}_j(x) s_{ij}(x) \prod_{1 \leq n \leq k} [1 - s_{ij}(x - nI_{ij})] dx \quad (21a)$$

or, for $\theta = 0$ as assumed by Pollock and Huso, just one term $k = 0$:

$$= m_{ij} \int_0^{I_{ij}} \bar{F}_j(x) s_{ij}(x) dx. \quad (21b)$$

Using $E[M_{ij}] = I_{ij} m_{ij}$, this leads to a variation on (8),

$$\hat{M}_{ij}^* = \frac{C_{ij} I_{ij}}{R_{ij}^*} \quad (22a)$$

where now R_{ij}^* is given by

$$R_{ij}^* \equiv \sum_{k=0}^{\infty} \theta^k \int_{kI_{ij}}^{(k+1)I_{ij}} \bar{F}_j(x) s_{ij}(x) \prod_{1 \leq n \leq k} [1 - s_{ij}(x - nI_{ij})] dx. \quad (22b)$$

This can be computed numerically for any specified proficiency function $s_{ij}(x)$ (like the logistic or exponential regressions of Eqn (20a) or Eqn (20b)) and any of the empirical or parametric removal distributions of Section (2.3) or Section (2.4.2).

4.3.1 An Efficient Computational Scheme

For exponentially diminishing proficiency $s_{ij}(t) = \exp(-a - bt)$ (which fits our data well, as shown in Figure (2)), R_{ij}^* is the sum

$$R_{ij}^* = I_{ij} \sum_{k=0}^{\infty} T_k$$

of terms

$$\begin{aligned} T_k &= \theta^k I_{ij}^{-1} \int_{kI_{ij}}^{(k+1)I_{ij}} \bar{F}_j(t) e^{-a-bt} \prod_{1 \leq n \leq k} [1 - e^{-a-b(t-nI_{ij})}] dt \\ &= \theta^k \int_0^1 \bar{F}_j((k+x)I_{ij}) e^{-a-bI_{ij}(x+k)} \prod_{0 \leq n < k} [1 - e^{-a-bI_{ij}(x+n)}] dx \end{aligned}$$

each expressible as the sum of 2^k terms of the form

$$Q_{kmn}^* = \theta^k (-1)^{m+1} \int_0^1 \bar{F}_j((k+x)I_{ij}) e^{-m(a+bI_{ij}x) - n b I_{ij}} dx \quad (23)$$

for suitable nonnegative integers k, m, n that can be enumerated recursively. The first few terms are

$$\begin{aligned} T_0 &= Q_{010}^* \\ T_1 &= Q_{111}^* + Q_{121}^* \\ T_2 &= Q_{212}^* + Q_{223}^* + Q_{222}^* + Q_{233}^* \\ T_3 &= Q_{313}^* + Q_{325}^* + Q_{324}^* + Q_{336}^* + Q_{323}^* + Q_{335}^* + Q_{334}^* + Q_{346}^* \end{aligned}$$

The integral in (23) is easily evaluated using Simpson's rule, or is available in closed form for Weibull removal with shape parameters $\alpha = 1$ (the exponential case) or $\alpha = \frac{1}{2}$ (very close to our estimated shape parameter $\hat{\alpha} = 0.4606$ for small birds). The truncation error from using only the first N terms $0 \leq k < N$ in (22b) is bounded by

$$0 \leq \text{Truncation Error} \leq \frac{\theta^N \bar{F}_j(N I_{ij}) \exp(-a - N b I_{ij})}{\max \{b I_{ij}, (1 - \theta e^{-b I_{ij}})\}}$$

For the parameter estimates in our study¹, a 40% error would be made using only $N = 1$ term (*i.e.*, neglecting bleed-through), but the error falls to 8% with $N = 2$ terms and below 0.5% with $N = 4$, indicating that a significant fraction of old carcasses were present from the immediately preceding period, but essentially none from more than three periods earlier. The values of R_{ij}^* and corresponding estimators $\hat{M}_{ij}^* = C_{ij}I_{ij}/R_{ij}^*$ with $\theta = 50\%$ were:

I_{ij}	2 d	7 d	14 d	28 d
R_{ij}^*	1.094	2.2224	2.9013	3.327
\hat{M}_{ij}^*	$1.8 C_{ij}$	$3.15 C_{ij}$	$4.8 C_{ij}$	$8.4 C_{ij}$

5 Interval Estimates for M_{ij} and m_{ij}

Each of the estimators in Sections (2) and (4) is of the form “ $\hat{M}_{ij} = \kappa C_{ij}$ ” for some $\kappa \geq 1$, proportional to the carcass count C_{ij} , because in each C_{ij} has either a binomial or a Poisson distribution with mean proportional to M_{ij} or to $m_{ij} = \mathbf{E}M_{ij}/I_{ij}$:

$$\mathbf{E}[M_{ij}] = m_{ij}I_{ij}, \quad \mathbf{E}[C_{ij}] = m_{ij}I_{ij}/\kappa, \quad \kappa \equiv \frac{I_{ij}}{s_{ij}R_{ij}} \text{ or } \frac{I_{ij}}{R_{ij}^*} \quad (24a)$$

where R_{ij} is given by Eqn (9a) and R_{ij}^* by Eqn (22b). In this Section we present Confidence Interval estimators for M_{ij} and m_{ij} .

5.1 Estimating M_{ij} when $\theta = 0$

If $\theta = 0$ then all of the C_{ij} carcasses discovered will be from the M_{ij} of the current period I_{ij} , with conditional distribution

$$C_{ij} \mid M_{ij} \sim \text{Bi}(M_{ij}, 1/\kappa).$$

If the constant κ is known precisely (if θ_j , I_{ij} , s_{ij} , and the removal distribution including its parameters are all known, for example), then an exact Confidence Interval for $M_{ij} \approx \hat{M}_{ij}^* = \kappa C_{ij}$ can be constructed as follows. For any chosen confidence level γ (like 0.90, for example), an exact $100\gamma\%$ Confidence Interval is given by

$$\gamma \leq \mathbf{P}\{\text{lo}(C_{ij}) \leq M_{ij} \leq \text{hi}(C_{ij})\} \quad (24b)$$

where the functions $\text{lo}()$ and $\text{hi}()$ are given for integers $c \geq 0$ by

$$\begin{aligned} \text{lo}(c) &= \sup \left\{ m \geq c : \text{pbeta}(1/\text{kap}; c+1, m-c) \leq \frac{1-\gamma}{2} \right\} \\ \text{hi}(c) &= \inf \left\{ m \geq c : \text{pbeta}(1/\text{kap}; c, m+1-c) \geq \frac{1+\gamma}{2} \right\} \end{aligned}$$

¹Maximum likelihood estimates were $\hat{\alpha} \approx 0.4606$, $\hat{r}_{ij} \approx 0.07944$, $\hat{a} = 1.017$ and $\hat{b} = 0.0777$

For large C_{ij} and moderate κ these are approximately

$$\text{lo}(c), \text{hi}(c) \approx \kappa \left[c \pm \sqrt{c} z \sqrt{(\kappa - 1)/\kappa} \right]$$

for $z := \Phi^{-1}\left(\frac{1+\gamma}{2}\right)$, so the CI widths are roughly proportional to \sqrt{c} for large c . They fall to zero for $c = 0$.

5.2 Estimating $\mathbf{E}M_{ij}$ when $0 \leq \theta \leq 1$

If $\theta > 0$ some carcasses discovered in a search may have arisen from fatalities in earlier time period— so it is possible to have a positive count $C_{ij} > 0$ even if $M_{ij} = 0$, making it challenging to find interval estimates for M_{ij} based only on the count C_{ij} from the current period. Under the assumption of near periodicity, however, even though the actual fatality counts M_{ij} will vary from period to period by chance, the *mean* fatality counts $\mathbf{E}M_{ij} = m_{ij}I_{ij}$ should be approximately constant. An exact 100 $\gamma\%$ Confidence Interval is given by

$$\gamma \leq \mathbf{P}\{\text{lo}(C_{ij}) \leq \mathbf{E}M_{ij} \leq \text{hi}(C_{ij})\} \quad (24c)$$

where the functions $\text{lo}()$ and $\text{hi}()$ are gamma distribution quantiles determined for integers $c \geq 0$ by the relations

$$(1 - \gamma)/2 = P(c, \text{lo}(c)/\kappa) \quad (1 + \gamma)/2 = P(c + 1, \text{hi}(c)/\kappa). \quad (24d)$$

In **R**, the solutions (with variables **c**, **gam** and **kap** for c , γ and κ respectively) are

$$\begin{aligned} \text{lo}(c) &= \text{kap} * \text{qgamma}((1-\text{gam})/2, c); \\ \text{hi}(c) &= \text{kap} * \text{qgamma}((1+\text{gam})/2, c+1); \end{aligned}$$

with similar formulas in MS Excel (where `GAMMA.INV(q,a,1)` takes the place of **R**'s `qgamma(q, a)`) or other environments. For example, with the parameter values used in the simulation shown in Figure 1, $R_{ij} = 0.49956$, $I_{ij}/t_{ij} = 10/15 = 0.6667$ and $s_{ij} = 0.3$, so $\kappa = I_{ij}/(R_{ij}s_{ij}) \approx 4.4483$. The proposed estimator is $\hat{M}_{ij}^* = \kappa C_{ij} \approx 4.45 \times C_{ij}$, and $\gamma = 90\%$ confidence interval estimates of M_{ij} (whose true value averaged $m_{ij}I_{ij} = 30$ in the simulation) for various count values of C_{ij} would be:

C_{ij}	lo	\hat{M}_{ij}^*	hi
0	0.00	0.00	13.33
1	0.23	4.45	21.10
2	1.58	8.90	28.01
3	3.64	13.35	34.49
4	6.08	17.79	40.72
5	8.76	22.24	46.77
6	11.62	26.69	52.68
7	14.61	31.14	58.49
8	17.71	35.59	64.21
9	20.89	40.04	69.86
10	24.13	44.48	75.45
11	27.44	48.93	80.99
12	30.80	53.38	86.49
13	34.21	57.83	91.94
14	37.65	62.28	97.36
15	41.13	66.73	102.74

This illustrates how imprecisely M_{ij} is determined by the counts C_{ij} (especially for low counts) even if α , θ_j , I_{ij} , s_{ij} , and t_{ij} are all known precisely. If any of these must be estimated, then the uncertainty about M_{ij} is greater.

Bayesian estimates and Credible Intervals are also available for conjugate gamma prior distribution $M_{ij} \sim \text{Ga}(a, b)$,

$$\mathbf{E}[M_{ij} | C_{ij}] = \frac{\kappa}{1 + \kappa b} [C_{ij} + a] = \frac{\hat{M}_{ij}^* + \kappa a}{1 + \kappa b} \quad (25a)$$

$$\gamma = \mathbf{P}\left\{ \text{lo}(C_{ij}) \leq M_{ij} \leq \text{hi}(C_{ij}) \right\} \quad (25b)$$

$$\text{lo}(c) = \text{qgamma}((1-\text{gam})/2, c+a) * \text{kap}/(1+\text{kap}*b); \quad (25c)$$

$$\text{hi}(c) = \text{qgamma}((1+\text{gam})/2, c+a) * \text{kap}/(1+\text{kap}*b); \quad (25d)$$

The reference or “noninformative” choice would be $a = \frac{1}{2}$, $b = 0$; more generally, experience in similar settings may suggest an appropriate “prior sample size” b and “prior sample sum” a . Note the same Confidence Intervals and Credible Intervals also apply to any of the estimators \hat{M}_{ij}^{EJ} , \hat{M}_{ij}^{H} , \hat{M}_{ij}^{S} , \hat{M}_{ij}^{H} , since they are special cases of \hat{M}_{ij}^* .

A Appendix: Notation Glossary

For convenience we collect here notation used in this document:

- \mathbf{g}_0 be the number of discoverable carcasses on the ground at the start of a period,
- \mathbf{g}_1 be the number of discoverable carcasses on the ground at the end of a period.

By periodicity we take to be the same for all periods, though they may vary with the turbine i (but to simplify the notation we ignore this). At turbine i and time period j , let:

C_{ij}	(count)	= number of carcasses counted,
I_{ij}	(search interval)	= number of days between searches,
M_{ij}	(mortality)	= true number of carcasses during interval,
m_{ij}	(mortality rate)	= daily average arrival rate of carcasses,
p_{ij}	(persistence probability)	= probability of remaining unremoved until next search,
r_{ij}	(rate parameter)	= common parameter for all removal distributions,
s_{ij}	(search proficiency)	= probability a carcass will be discovered,
t_{ij}	(mean persistence)	= average number of days a carcass remains unremoved,
α	(shape parameter)	= common parameter for all removal distributions,
θ_j	(periodicity)	= fraction of undiscovered carcasses that remain discoverable,
τ	(persistence time)	= number of days a carcass remains unremoved,
q_j	$(\theta_j(1 - s_{ij}))$	= probability undiscovered but still discoverable,
$\bar{F}_j(t)$	(survival function)	= $\mathbb{P}[\tau > t]$,
$Q_j(t)$	(ISF)	= $\mathbb{E}[\tau \wedge t] = \int_0^t \bar{F}_j(x) dx$,

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APPENDIX B: A New Survey Method for Detection Probability Trials for Partial Periodic Estimator

Introduction

This study identified and explained major influences on detection probability for periodic searches of bird and bat fatalities. These discoveries led to the creation of a new Partial Periodic Estimator (Appendix A), which requires modifications to the traditional survey methodology. The following lays out the framework for wind energy fatality monitoring surveys and detection probability trials that support the new estimator and account for the major influences on detection probability.

1. Time dependent carcass persistence and searcher proficiency: It has been well documented that carcass persistence is dependent on carcass age, and this study shows that search proficiency is also dependent on carcass age.
2. Carcasses can persist through multiple search intervals, allowing for multiple detection events: Some of the previous fatality estimation equations (*e.g.*, Pollock and Huso) do not account for the common occurrence of carcasses being deposited in one search interval that persist into subsequent intervals and are detected at a later date. The Partial Periodic Estimator measures this “bleed-through” process with a new term, “theta,” which is the proportion of undiscovered carcasses that remain discoverable.
3. Other covariates such as vegetation height can also have strong influences on detection probability.

Preliminary Scavenger Removal Trial

Before a main study begins, a preliminary traditional 60-day scavenger removal trial is required to estimate the ballpark mean persistence of carcasses (bats, small birds, and large birds) and variation in removal times. The recommended main study search interval is equal to the shortest mean persistence of the three carcass types placed. The recommended main detection probability trial length is three times the mean persistence of the longest persisting carcass type. The number of carcasses used in the main detection probability trial for each size category should be based on the variation of removal times.

Main Study Detection Probability Trial

Carcass Placement

Carcass placement timing should occur to simulate the assumed steady random rate of deposit. Carcasses should be placed at random positions in a search area to account for covariates such as vegetation height and slope. Carcasses should be marked to distinguish them as trial carcasses and not true fatalities. Carcasses should be mapped with sub-meter accurate Global Positioning System (GPS) receivers, or their positions should be cryptically marked to help a project field manager certify their presence while keeping field technicians blind to their

presence. Main study detection probability trials should occur at least once per season, and the trial length should be equal to three times the mean persistence of the longest persisting carcass type in the preliminary scavenger removal trial. All carcasses should be placed at monitored wind turbines.

Integrated Carcass Persistence and Proficiency Detection Events

For any given carcass and search, the probability of persistence and detection (searcher proficiency) are both time dependent and dependent on one another. This makes it highly effective and desirable to measure these outcomes together in an integrated trial, rather than in two independent trials.

After placing trial carcasses strategically at monitored wind turbines, carcass persistence and searcher proficiency needs to be measured.

To establish carcass persistence, a traditional scavenger removal trial schedule of carcass checks is recommended for all trial carcasses –the project field manager checks carcasses every day for the first week, every three days for the next two weeks, and then every seven days until all carcasses are removed or the end of the trial is reached. In addition to the traditional schedule of carcass checks, supplemental carcass checks should occur for trial carcasses on search days. Note that many of the supplemental carcass checks will overlap with the traditional schedule of carcass checks and will not require extra effort. Carcass checks of trial carcasses on the day of searches should be conducted after field technicians complete their searches to maintain the searchers' blindness.

To establish searcher proficiency, field technicians record all marked carcasses they detect while conducting their standard scheduled searches. They should be instructed not to disturb these marked carcasses; they are left in place for future project field manager persistence carcass checks. Because the project field manager conducts carcass checks of trial carcasses on search days, the true persistence status of those trial carcasses is known; therefore negative searcher detections can be interpreted as either a searcher's miss of a persisting trial carcass or that the trial carcass was removed by scavengers.

Integrating the carcass persistence and searcher proficiency trials can simultaneously produce time dependent carcass persistence and searcher proficiency functions for the same set of trial carcasses.

Search Interval Bleed-through of Carcasses: Theta

The final term that needs to be measured for the Partial Periodic Estimator is theta, the fraction of undiscovered carcasses that remain discoverable over time through multiple search intervals. Because trial carcasses are placed to simulate a random steady state of deposit at monitored wind turbines and the persistence and detection of trial carcasses are tracked, the number of trial carcasses that are not detected and not removed in one interval that persist to be possibly detected in a subsequent interval can be measured.

Collateral Data and Advantages to the New Method

Because a preliminary persistence trial is conducted first, proper trial carcass sample size, trial length, and search interval can be established for the main monitoring program ahead of time. This will introduce an evidence-based approach rather than guessing or using a “rule-of-thumb” to establish these aspects of the program.

The data collected from the new method can be used to source estimates for all four traditional fatality estimation equations reviewed in this study. The traditional persistence carcass check schedule is conserved, and the traditional simple initial fresh carcass searcher proficiencies can be extracted from the initial detection outcome of this method. This can allow for easy comparisons of estimator results to compare to previous studies that used other estimators. In addition, a remarkably simple empirical estimator is also sourced by the data collected and can be used as an independent check on the Partial Periodic Estimator. The number of total searcher-detected trial carcasses divided by the number of placed trial carcasses should be equal to (or close to) the overall detection probability derived by the Partial Periodic Estimator. This is because the effects of the integrated time dependent probabilities of carcasses persistence and searcher proficiency as well as the bleed-through theta mechanism are implicit in the proportional detection outcomes of this new method.

Overall, this new method and estimator are much more sensitive to the major influences that affect detection probability, reducing bias and improving the predictive power of estimating the impacts of wind turbines on wildlife.

APPENDIX C: Data Dictionary & Data Fields Used for Recording Carcasses

Table C-1: Data Dictionary

Variable	Description	Units
ID	Unique record identifier for all data rows	Number
Date	Date that a status check or search took place	Julian date format
String	Unique identifier for a collection of turbines where trial carcasses were placed and searches occurred	Unique number
Species	The species or unknown species determination (ex UNRA, unknown raptor)	AOU species code
Photo	Unique identifier per photo	Photo number
Sex	The sex determination of trial carcasses, if known	U=unknown; M=Male; F=Female
Age	The age class of the trial carcass, if known	A=adult; J=juvenile; U=unknown
Class	The group status of trial carcass, Bird or Bat	Bird or Bat
Grid_Cell	The dominant grid cell that the carcass occupies on specified date	Alpha-numeric map key
PositionID	ID at time of search, based on last known position	Carcass_ID + position modifier
AssignedID	ID after QA and analysis, may combine several unknown or found IDs	Carcass_ID
PID	An identifying number for the project field manager who conduct the status check. Searcher that conduct the search	See data file for codes
Person	Project field manager or field technician	Name
SearchDay	Does record represent a day when searchers were present	Yes / No
DetectionStatus	The detection outcome generated by a status check or search	P = placement of carcass; F = found carcass; NF = a not found carcass; NC = a not checked carcass position (only after many prior checks, and assurance that carcass has been removed)

Variable	Description	Units
DetectionType	The type of detection (if detected)* <i>* If a specific carcass was ever detected during the study, it was considered a detection.</i>	S: Found during standard status check without additional effort; F: Found during flushing search around last known location of carcass; I: Found incidentally at unknown position without systematic search; 0: Found but not enough evidence to be considered fatality; M: Found carcass due to Marker (FM) or Did Not Find the Marker (NFM)
PositionPresence	Indicator of carcass presence at time of search	1=present; 0=absent
AssignedPresence	Indicator of carcass presence after analysis and QA	1=present; 0=absent
Veg_HT	Vegetation height	Inches
ScavengerIndex	A subjective index of the carcass "attraction" to a scavenger on a day	Index 1: Fresh carcass and very attractive for removal/scavenging; Index 2: Partially scavenged or decayed carcass , moderately attractive for removal/scavenging; Index 3: Completely scavenged or decayed (no remaining edible or attractive tissue), low attraction for removal/scavenging
GPSMarkID	Garmin record ID; allows sync with latitude and longitude	Number
Latitude	Position where carcass found during search	GPS Lat
Longitude	Position where carcass found during search	GPS Long
Note	Any field notes made by searcher or project field manager	Text
BlockNum	Block ID: contains multiple strings searched in a consistent time period	Ranges from 1 – 4
DistanceSighted	Distance from searcher to found carcass	Meters

Variable	Description	Units
TrialCarcass	Indicator of a trial carcass placed at t=0	Yes / No
TemperatureHighF	Daily high temperature	Degrees F
TemperatureAvgF	Daily average temperature	Degrees F
TemperatureLowF	Daily low temperature	Degrees F
DewpointHighF	Daily high dewpoint	Degrees F
DewpointAvgF	Daily average dewpoint	Degrees F
DewpointLowF	Daily low dewpoint	Degrees F
HumidityHigh	Daily high humidity	Percent
HumidityAvg	Daily average humidity	Percent
HumidityLow	Daily low humidity	Percent
PressureMaxIn	Daily maximum pressure	mmBars
PressureMinIn	Daily minimum pressure	mmBars
WindSpeedMaxMPH	Daily maximum wind speed	Miles per hour
WindSpeedAvgMPH	Daily average wind speed	Miles per hour
GustSpeedMaxMPH	Daily maximum wind gust speed	Miles per hour
PrecipitationSumIn	Daily total precipitation	Inches
RELEV	Elevation (feet) of nearest grid cell at the ridge crest	Feet
VELEV	Elevation (feet) of nearest grid cell at the valley bottom	Feet
DELTAELV	Change in elevation (feet) between nearest ridge crest and nearest valley bottom. Measure of slope size	Feet
TOTDIST	Total horizontal distance (feet) between nearest valley bottom and nearest ridge crest. Measure of slope size.	Feet
RDIST	Horizontal distance (feet) between grid cell and nearest ridge crest	Feet
VDIST	Horizontal distance (feet) between grid cell and nearest valley bottom	Feet
DEMELV	Elevation (feet) of target grid cell centroid, according to digital elevation model	Feet
ASPECT	Degrees from true north toward which the grid cell faces	Degrees
SLOPE	Percentage slope of grid cell, determined by trend with nearest grid cell in the uphill direction and with the nearest grid cell in the downhill direction. Measures local slope.	Percent

Variable	Description	Units
SLPBIN	Slope values aggregated into bins	Percent
VPLYDIST	Horizontal distance (feet) between grid cell and nearest grid cell along boundary of the closest valley bottom polygon.	Feet
VPLYELV	Vertical distance (feet) between grid cell and nearest grid cell along boundary of the closest valley bottom polygon.	Feet
RPLYDIST	Horizontal distance (feet) between grid cell and nearest grid cell along boundary of the closest ridge top polygon.	Feet
RPLYELV	Vertical distance (feet) between grid cell and nearest grid cell along boundary of the closest ridge top polygon.	Feet
Within slope hazard zone?	Whether grid cell occurs within a ridge saddle, break in slope, or other slope feature determined to be more often used by flying raptors. This determination was judgment based, and not the product of modeling.	1=yes; 0=no
Gross slope	Average slope from nearest valley bottom to nearest ridge crest, measured as ratio of elevation difference and total slope distance.	Ratio (%)
Distance ratio	Ratio of horizontal distance (feet) between grid cell and nearest valley bottom and of distance between grid cell and nearest ridge crest. Values of #DIV/0! in this ratio occurred for grid cells at the ridge crest; repairs were left to the analyst.	Ratio (%)
Elevation ratio	Ratio of vertical distance (feet) between grid cell and nearest valley bottom and of vertical distance between grid cell and nearest ridge crest. Values of #DIV/0! in this ratio occurred for grid cells at the ridge crest; repairs were left to the analyst.	Ratio (%)

Source: EcoStat, Inc.

Table C-2: Master Data Fields Used in Recording Carcasses Found

Recorded Master Data Fields
Record ID: Unique record identifier for all data rows.
Date: Date that a status check or search took place.
String: Unique identifier for a collection of turbines that trial carcasses were placed and searches occurred.
Species: The species or unknown species determination (ex UNRA, unknown raptor).
Sex: The sex determination of trial carcasses, if known.
Age: The age class of the trial carcass, if known.
Class: The group status of trial carcass, Bird or Bat.
Grid Cell: The dominant grid cell that the carcass occupied on specified date.
Carcass ID: The unique identifier for marked placed trial carcasses, naturally detected carcasses, and unknown marked carcasses.
Assigned ID: The assigned carcass ID for unknown marked carcasses based on proximity to known carcass ID positions and presence status.
P_ID: Identifying number for project field manager who conducted status check, and searcher who conducted search.
Search Outcome: The search outcome, whether a carcass was detected on a day Yes/No.
Presence: The known presence of a carcass on a day Yes/No.
Vegetation height: The vegetation height measure at the position of the carcass.
Scavenger Index: A relative index of carcass condition.
Index 1: A fresh carcass.
Index 2: A partially scavenged or decayed carcass.
Index 3: A completely scavenged or decayed (no remaining edible tissue).
Recorded Master Data Fields
Topo: A topographical feature that the carcass position occupied.
Detection status: The detection outcome generated by a status check or search.
P: Placement of a trial carcass
F: Carcass found
NF: Carcass not found
NC: Carcass position not checked
Detection type: The type of detection (if detected).
S: Found during a standard status check without additional effort.
F: Found during a flushing search around the last known location of a carcass.
I: Found incidentally at an unknown position without a systematic search.
0: Found but not enough evidence to be considered a fatality.
M: Found carcass due to the Marker (FM) or Did Not Find the Marker (NFM).

Source: EcoStat, Inc.

Table C-3: Data Collected by Project Field Managers for Unknown Carcasses

Date	mm/dd/yyyy
Project Field Manager	Project field manager initials.
String	String number.
Start and End Time	24 hour time. Time when the field technician arrived at the string and left after completing the search.
Grid Cell	Alphanumeric cell address indicating the position of the fatality remains.
Species	Project field manager's best understanding of species identification.
Nearest Turbine	The closest complete turbine to the evidence of fatality.
Distance	Distance (in meters) from evidence of fatality to Nearest Turbine.
Bearing	Compass bearing from the Nearest Turbine to the evidence of fatality.
Latitude Longitude	GPS NAD 24 CONUS hddd.ddddd
Carcass sign	One or more code can be entered. Coded categories of carcass sign for evidence of fatality: F = 10 or more feathers W = partial or intact wing or wings T = partial or intact tail PB = body parts or partial body WB = complete whole body H = partial skull or complete head
Photo number	Camera letter and photo numbers.
Vegetation height	The vegetation height (in inches) at the position of the evidence of fatality.
Marked	Yes or No indicating whether the fatality legs and wings were taped or whether the flight feather (wing and tail) were clipped.
Carcass ID	If the legs were taped, the number indicated was recorded.
Scavenger Index	A relative rating of carcass condition: 1 – Fresh 2 – Partially scavenged or decayed 3 – Completely scavenged (feather spots or bones) or very decayed
Notes	

Source: EcoStat, Inc.

APPENDIX D: Figures Illustrating Biases for Equations in the Current Literature

Robert L. Wolpert

The four estimating equations considered here all represent quite similar attempts to estimate the actual number of avian fatalities in a specified area during each of a sequence of time intervals from counts of carcasses. For a variety of reasons some carcasses may not be counted: some may have been removed by scavengers, some may have fallen outside the search region, and searchers may fail to see some carcasses. The four equations differ in the assumptions they make in order to adjust for these missing carcasses.

Two of the estimation equations, those of Erickson, Johnson, *et al.* and of Shoenfeld, assume that search teams will find both “new” carcasses (those killed during the current time period) and “old” ones (those killed during earlier periods, but not removed by scavengers or search teams). Old and new carcasses are assumed to be equally likely to be removed by scavengers, and equally likely to be discovered in a subsequent searches. These estimators will *under-estimate* true mortality if these assumptions are wrong.

Conversely the other two estimation equations, those of Pollock and of Huso, begin with the assumption that *all* carcasses counted are new (*i.e.*, died during the current time interval). Both will *over-estimate* true mortality if this is wrong.

Shoenfeld’s estimator always exceeds that of Erickson, Johnson, *et al.*, because the latter assume (incorrectly, in practice) that search teams do not remove carcasses. Huso’s estimator is identical¹ to a special case of Pollock’s: the case in which scavengers are assumed to remove fresh carcasses and old ones at the same rates (technically, that the “persistence time” before scavengers remove a carcass have “exponential” probability distributions). Pollock’s estimator does not require that assumption. For exponential persistence times, the estimators of Erickson, Johnson, *et al.*, Shoenfeld, Pollock, and Huso are ordered consistently

$$\hat{M}_{ij}^{EJ} < \hat{M}_{ij}^S < \hat{M}_{ij}^P \leq \hat{M}_{ij}^H$$

All four give similar estimates when the interval between searches is long compared to mean carcass persistence times, but differences among them are larger when searches are made more frequently to reduce the loss of carcasses to scavenging. With frequent searches, \hat{M}_{ij}^P and \hat{M}_{ij}^H can be as much as three or four times larger than \hat{M}_{ij}^S for small birds. The key issue, then, to guide the choice of estimators, is:

What fraction of carcasses missed by a search team might still be discovered as “old” carcasses in a later search?

If that fraction is 100% then Shoenfeld’s estimator \hat{M}_{ij}^S is most accurate on average if search teams remove the carcasses they discover, and Erickson & Johnson’s \hat{M}_{ij}^{EJ} if they don’t.

If that fraction is 0% then Pollocks’s estimator \hat{M}_{ij}^P is most accurate on average, with the side benefit that it does not require the “exponential distribution” assumption.

If that fraction is somewhere between 0% and 100%, then some sort of compromise between \hat{M}_{ij}^S and \hat{M}_{ij}^P is called for. Such a compromise is proposed and described in Appendix A, *A New Equation for Estimating Avian Mortality Rates*.

¹Except that Huso’s estimator is inflated by about 1% in the rare case when intervals between consecutive searches are more than 4.6 times the average length of time before scavengers remove a carcass.

Figures Illustrating Equation Biases

Figures 1–6 below show eight-week simulations of carcass arrivals and removals by scavengers as stair-step curves increasing at each arrival and decreasing at each removal, with searches at specified intervals from two to 56 days. Carcasses discovered and removed are indicated by downwards pointing red arrows; expected numbers of carcasses are indicated by smooth blue curves.

Figures 1–3 assume exponential distributions for persistence times (so scavengers remove fresh and older carcasses at the same rates), while Figures 4–6 assume Weibull removal distributions with parameter values suggested by our data.

Figures 1, 4 assume that only “new” carcasses can be discovered, so each curve begins each search period with zero carcasses. This assumption underlies Pollock’s and Huso’s estimators, so their bias is zero in the exponential persistence case, Figure 1 (and, for Pollock, also for Weibull persistence, Figure 4).

Figures 3, 6 assume that 100% of old carcasses remain discoverable, so each curve begins at the point of the red arrow (indicating that carcasses disappear only because of their discovery by search teams). This assumption underlies Erickson, Johnson, *et al.*’s and Shoenfeld’s estimators, so Shoenfeld has no bias in Figure 3. Erickson, Johnson, *et al.* still underestimate M_{ij} there because of their assumption that search teams don’t remove carcasses.

Finally, figures 2, 5 take the compromise position that (on average) 50% of undiscovered carcasses will remain discoverable; typically here Erickson, Johnson, *et al.*’s and Shoenfeld’s estimators will underestimate, while Pollock’s and Huso’s will overestimate.

Below each of these thirty plots is a table giving the bias (as a percentage of the truth) for each of the four estimators (or five, for Weibull distributions, where results for both exponential and Weibull versions of Pollock’s estimator are reported).

All the biases are smaller for long search intervals (at the top of each figure) and greater for shorter ones (at the bottom of each figure). Huso’s estimator is identical to the exponential version of Pollock’s, and so has the same bias in every case. The new estimator described in Appendix A, *A New Equation for Estimating Avian Mortality Rates*, has zero bias in all of these cases.

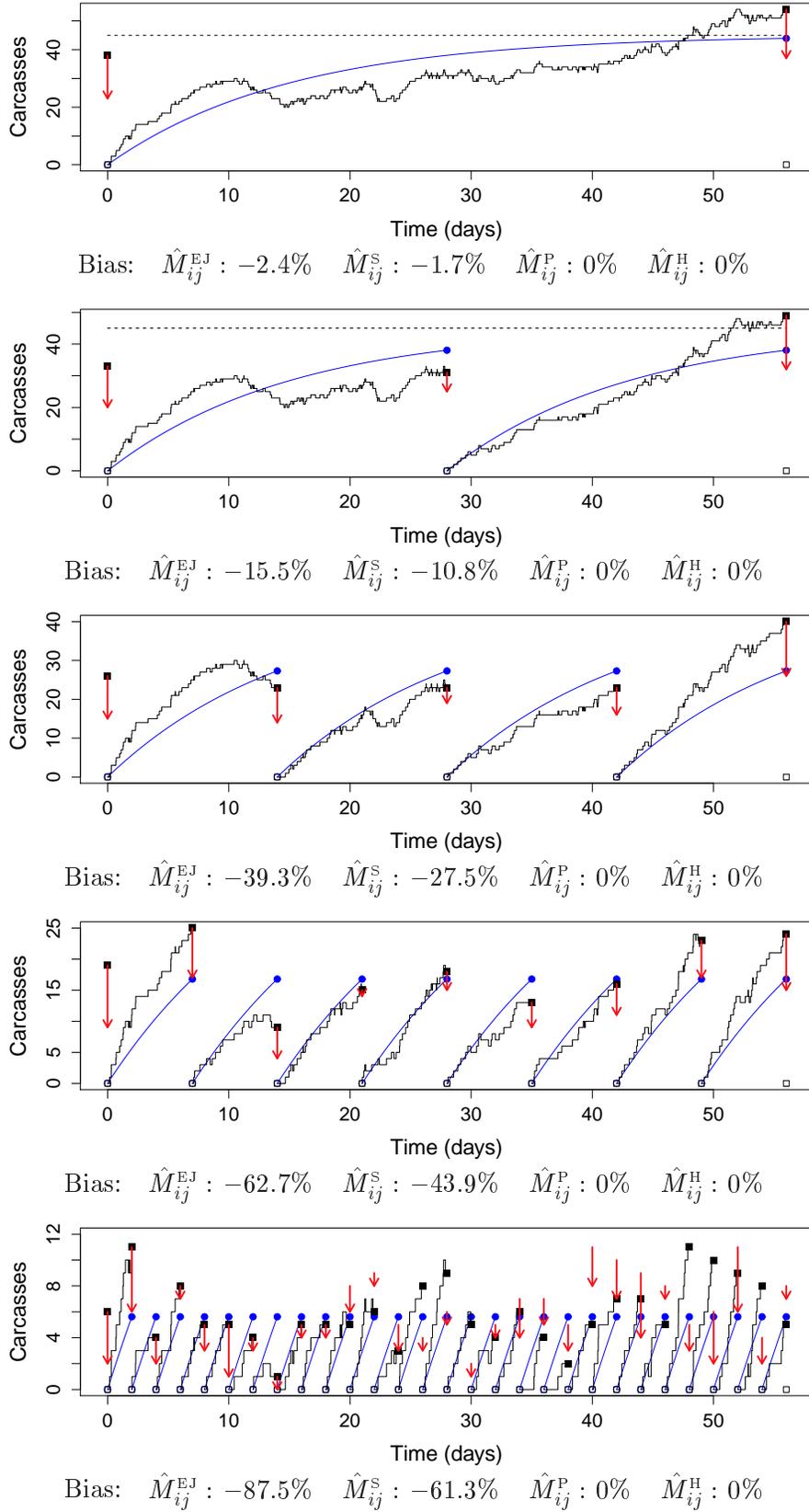


Figure 1: Exponential Persistence, Full Degradation: $\alpha = 1, \theta = 0$

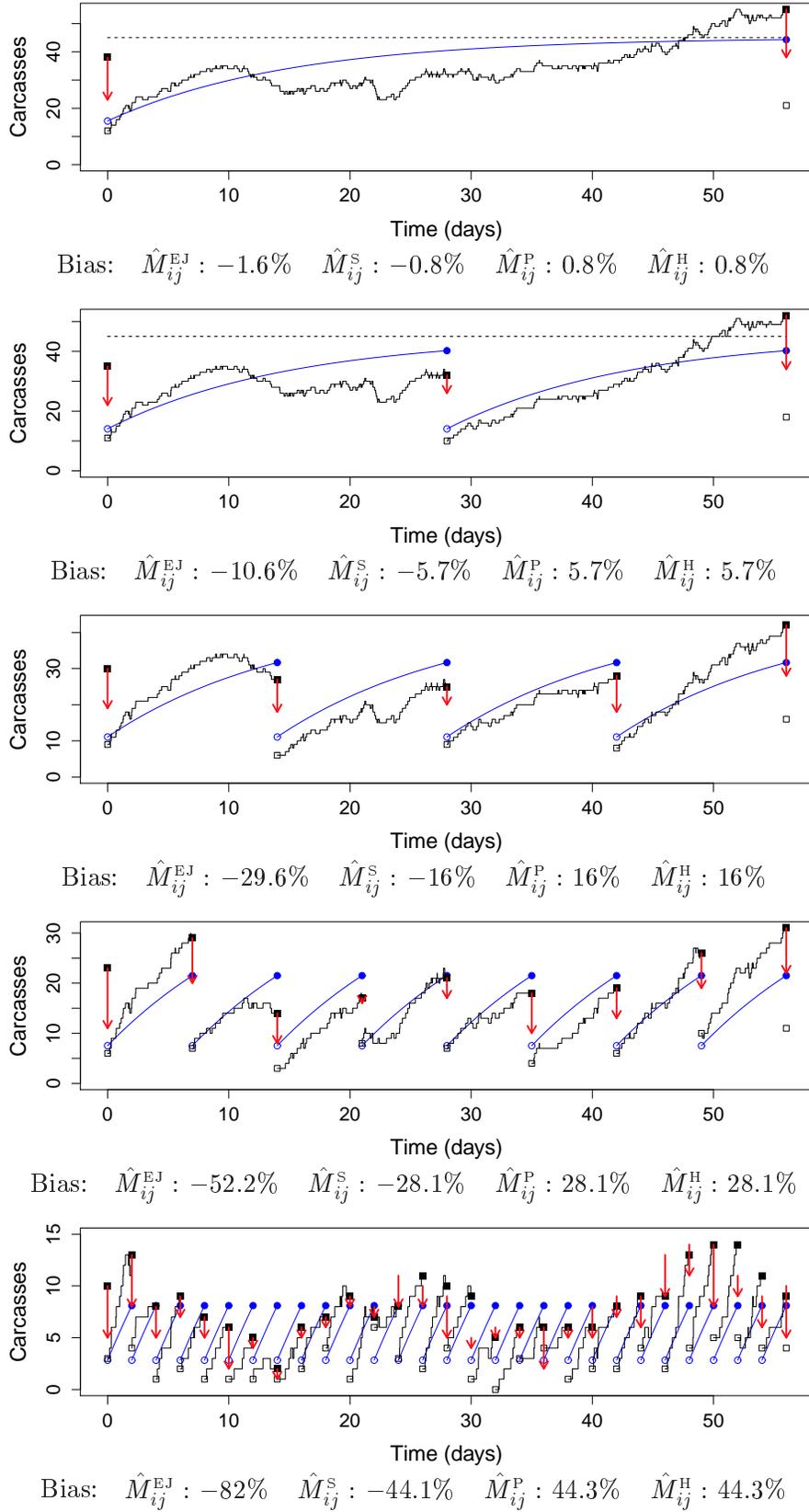
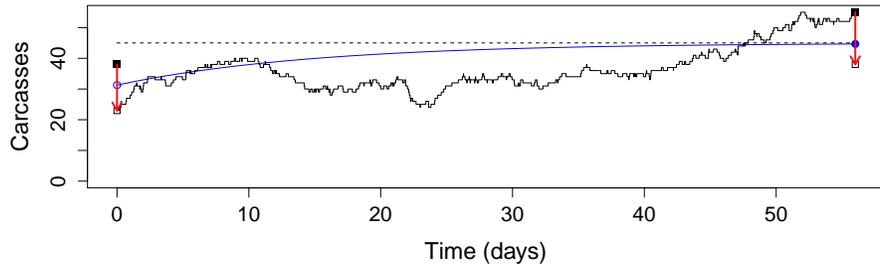
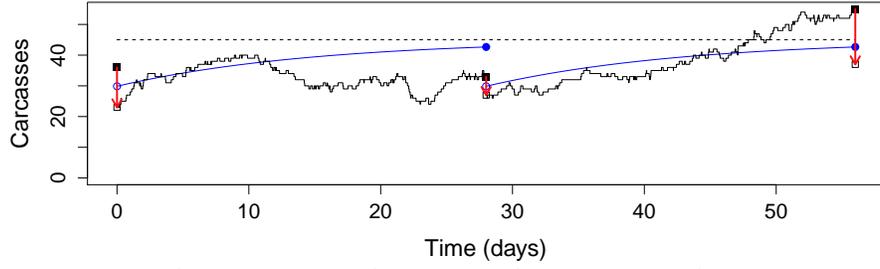


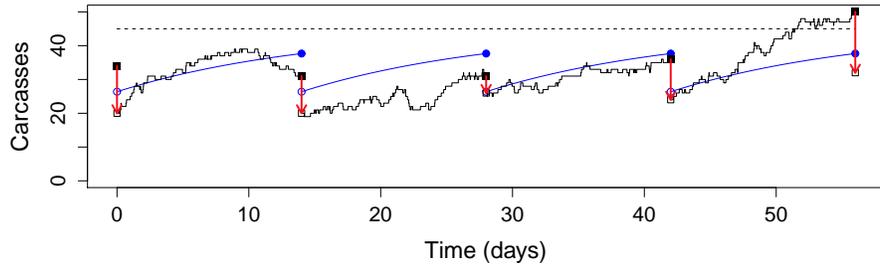
Figure 2: Exponential Persistence, Half Degradation: $\alpha = 1, \theta = 0.5$



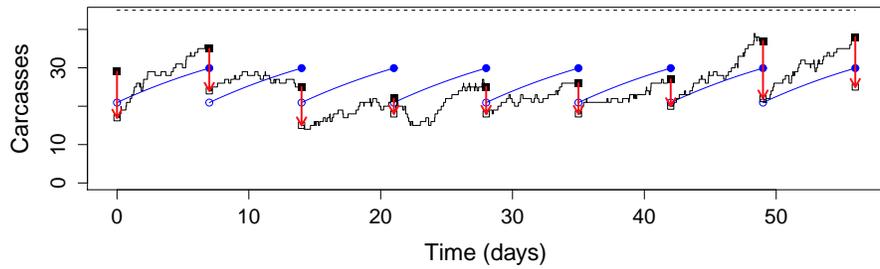
Bias: $\hat{M}_{ij}^{EJ} : -0.7\%$ $\hat{M}_{ij}^S : 0\%$ $\hat{M}_{ij}^P : 1.7\%$ $\hat{M}_{ij}^H : 1.7\%$



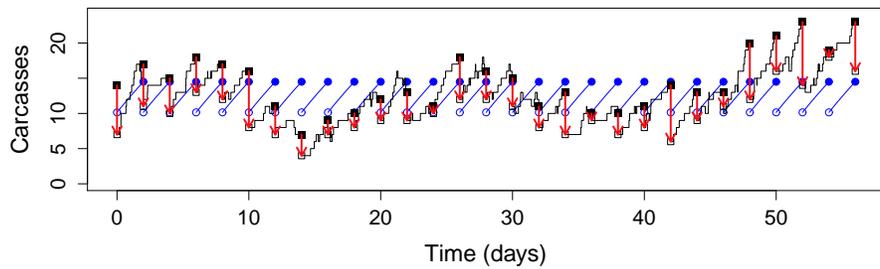
Bias: $\hat{M}_{ij}^{EJ} : -5.2\%$ $\hat{M}_{ij}^S : 0\%$ $\hat{M}_{ij}^P : 12.1\%$ $\hat{M}_{ij}^H : 12.1\%$



Bias: $\hat{M}_{ij}^{EJ} : -16.3\%$ $\hat{M}_{ij}^S : 0\%$ $\hat{M}_{ij}^P : 38\%$ $\hat{M}_{ij}^H : 38\%$

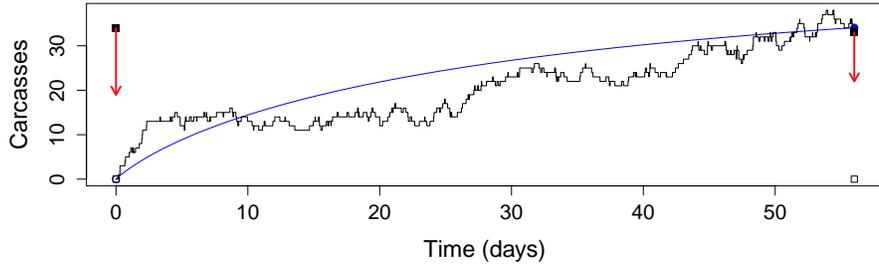


Bias: $\hat{M}_{ij}^{EJ} : -33.5\%$ $\hat{M}_{ij}^S : 0\%$ $\hat{M}_{ij}^P : 78.2\%$ $\hat{M}_{ij}^H : 78.2\%$

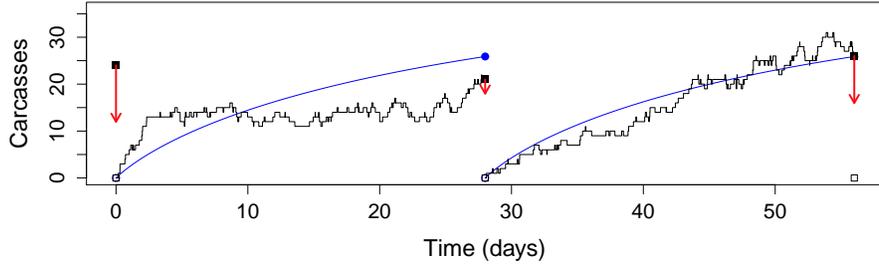


Bias: $\hat{M}_{ij}^{EJ} : -67.8\%$ $\hat{M}_{ij}^S : 0\%$ $\hat{M}_{ij}^P : 158.2\%$ $\hat{M}_{ij}^H : 158.2\%$

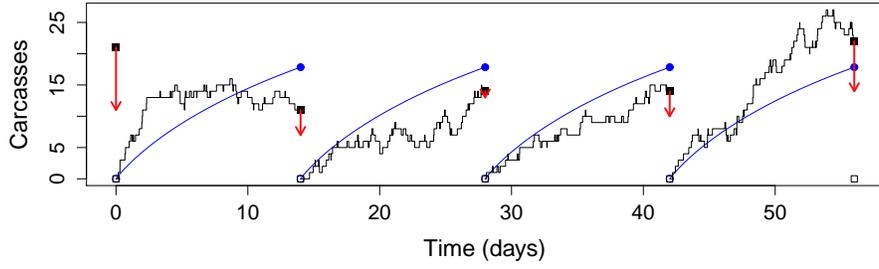
Figure 3: Exponential Persistence, No Degradation: $\alpha = 1, \theta = 1$



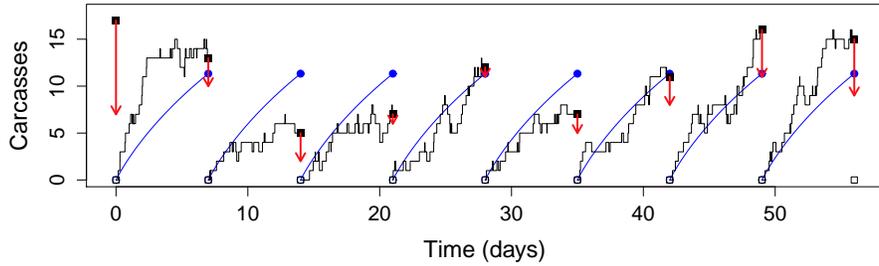
Bias: $\hat{M}_{ij}^{EJ} : -24.3\%$ $\hat{M}_{ij}^S : -23.7\%$ $\hat{M}_{ij}^{P:E} : -22.4\%$ $\hat{M}_{ij}^{P:W} : 0\%$ $\hat{M}_{ij}^H : -22.4\%$



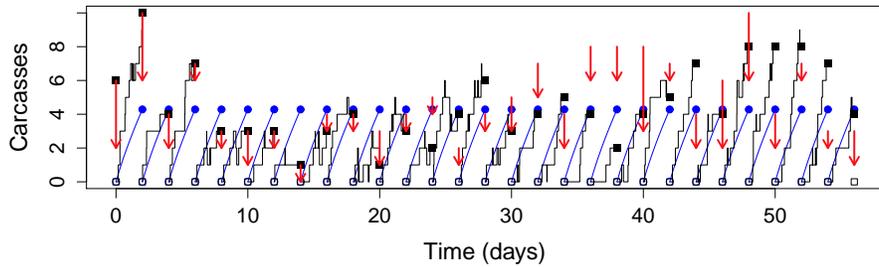
Bias: $\hat{M}_{ij}^{EJ} : -42.5\%$ $\hat{M}_{ij}^S : -39.3\%$ $\hat{M}_{ij}^{P:E} : -31.9\%$ $\hat{M}_{ij}^{P:W} : 0\%$ $\hat{M}_{ij}^H : -31.9\%$



Bias: $\hat{M}_{ij}^{EJ} : -60.4\%$ $\hat{M}_{ij}^S : -52.6\%$ $\hat{M}_{ij}^{P:E} : -34.7\%$ $\hat{M}_{ij}^{P:W} : 0\%$ $\hat{M}_{ij}^H : -34.7\%$

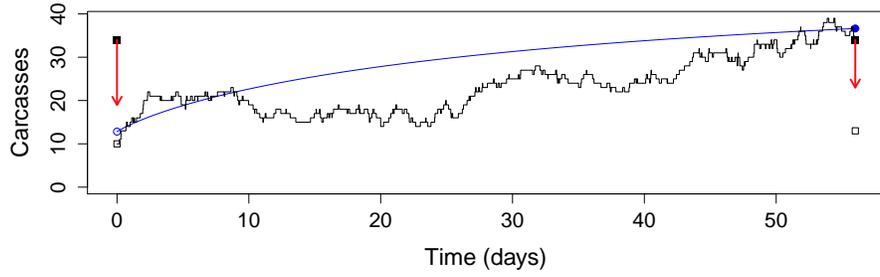


Bias: $\hat{M}_{ij}^{EJ} : -74.8\%$ $\hat{M}_{ij}^S : -62.1\%$ $\hat{M}_{ij}^{P:E} : -32.5\%$ $\hat{M}_{ij}^{P:W} : 0\%$ $\hat{M}_{ij}^H : -32.5\%$

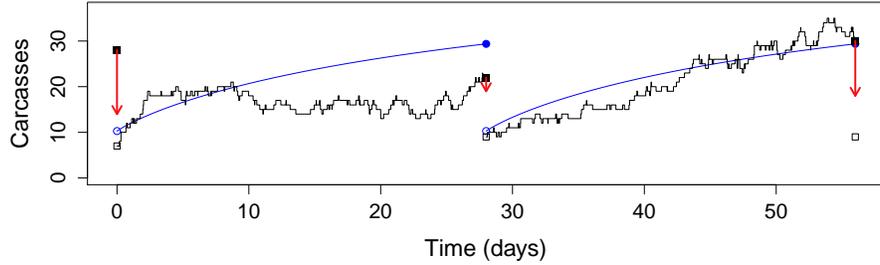


Bias: $\hat{M}_{ij}^{EJ} : -90.5\%$ $\hat{M}_{ij}^S : -70.5\%$ $\hat{M}_{ij}^{P:E} : -23.7\%$ $\hat{M}_{ij}^{P:W} : 0\%$ $\hat{M}_{ij}^H : -23.7\%$

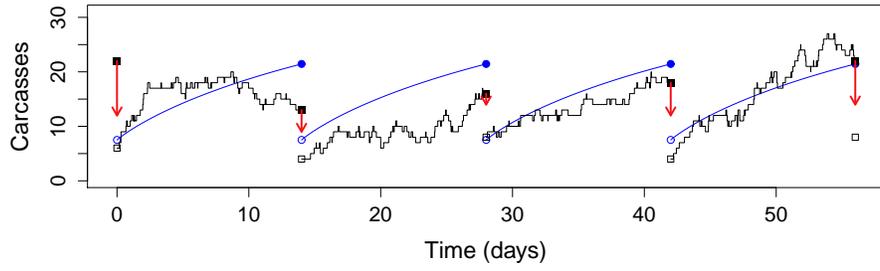
Figure 4: Weibull Persistence, Full Degradation: $\alpha = 0.5$, $\theta = 0$



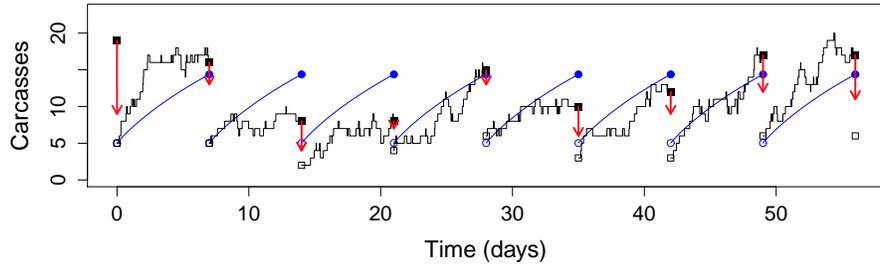
Bias: $\hat{M}_{ij}^{EJ} : -18.6\%$ $\hat{M}_{ij}^S : -18\%$ $\hat{M}_{ij}^{P:E} : -16.6\%$ $\hat{M}_{ij}^{P:W} : 7.5\%$ $\hat{M}_{ij}^H : -16.6\%$



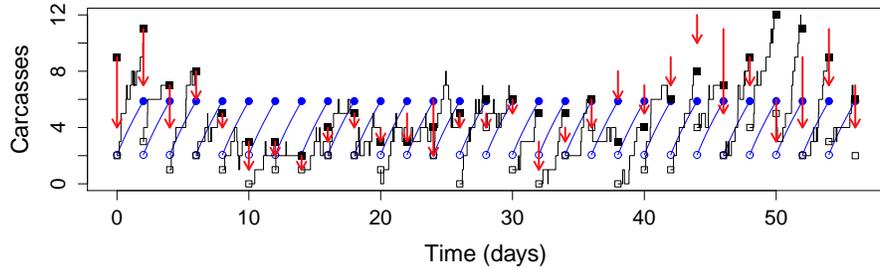
Bias: $\hat{M}_{ij}^{EJ} : -34.7\%$ $\hat{M}_{ij}^S : -31.1\%$ $\hat{M}_{ij}^{P:E} : -22.8\%$ $\hat{M}_{ij}^{P:W} : 13.5\%$ $\hat{M}_{ij}^H : -22.8\%$



Bias: $\hat{M}_{ij}^{EJ} : -52.3\%$ $\hat{M}_{ij}^S : -43.1\%$ $\hat{M}_{ij}^{P:E} : -21.4\%$ $\hat{M}_{ij}^{P:W} : 20.2\%$ $\hat{M}_{ij}^H : -21.4\%$

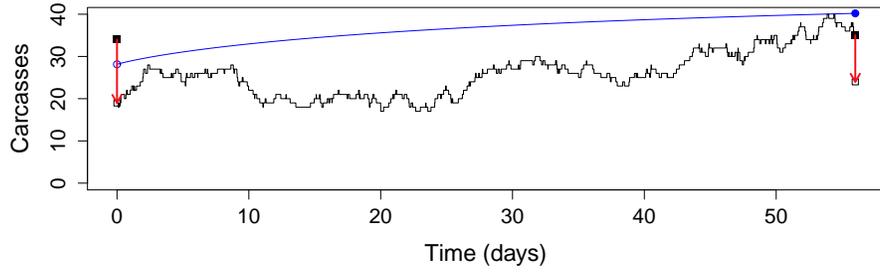


Bias: $\hat{M}_{ij}^{EJ} : -68\%$ $\hat{M}_{ij}^S : -51.9\%$ $\hat{M}_{ij}^{P:E} : -14.3\%$ $\hat{M}_{ij}^{P:W} : 26.9\%$ $\hat{M}_{ij}^H : -14.3\%$

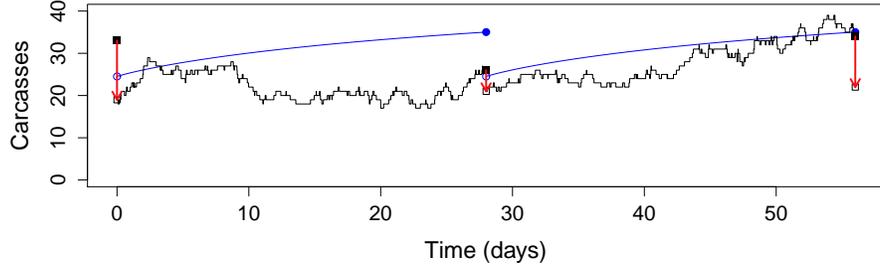


Bias: $\hat{M}_{ij}^{EJ} : -86.9\%$ $\hat{M}_{ij}^S : -59.4\%$ $\hat{M}_{ij}^{P:E} : 4.8\%$ $\hat{M}_{ij}^{P:W} : 37.5\%$ $\hat{M}_{ij}^H : 4.8\%$

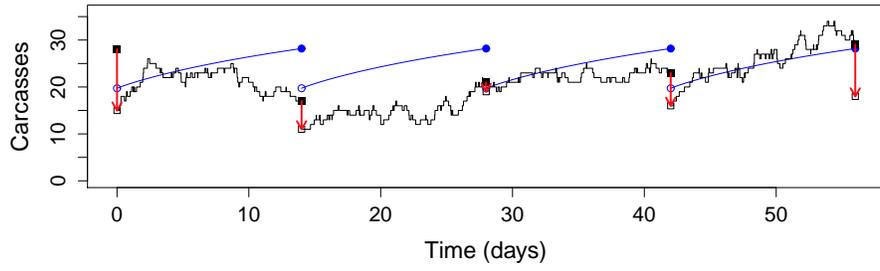
Figure 5: Weibull Persistence, Half Degradation: $\alpha = 0.5$, $\theta = 0.5$



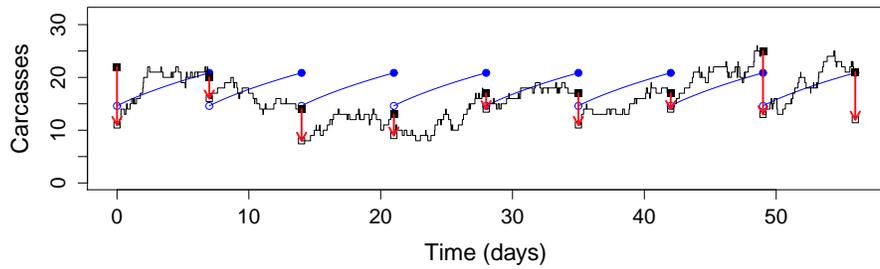
Bias: $\hat{M}_{ij}^{EJ} : -10.7\%$ $\hat{M}_{ij}^S : -10\%$ $\hat{M}_{ij}^{P:E} : -8.5\%$ $\hat{M}_{ij}^{P:W} : 18\%$ $\hat{M}_{ij}^H : -8.5\%$



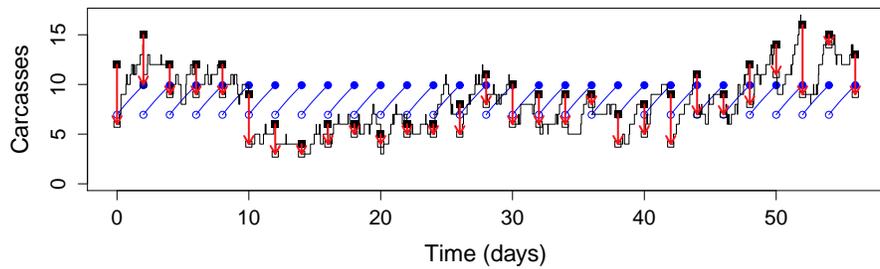
Bias: $\hat{M}_{ij}^{EJ} : -22.2\%$ $\hat{M}_{ij}^S : -17.9\%$ $\hat{M}_{ij}^{P:E} : -7.9\%$ $\hat{M}_{ij}^{P:W} : 35.3\%$ $\hat{M}_{ij}^H : -7.9\%$



Bias: $\hat{M}_{ij}^{EJ} : -37.4\%$ $\hat{M}_{ij}^S : -25.2\%$ $\hat{M}_{ij}^{P:E} : 3.2\%$ $\hat{M}_{ij}^{P:W} : 58\%$ $\hat{M}_{ij}^H : 3.2\%$



Bias: $\hat{M}_{ij}^{EJ} : -53.6\%$ $\hat{M}_{ij}^S : -30.3\%$ $\hat{M}_{ij}^{P:E} : 24.3\%$ $\hat{M}_{ij}^{P:W} : 84.1\%$ $\hat{M}_{ij}^H : 24.3\%$



Bias: $\hat{M}_{ij}^{EJ} : -77.9\%$ $\hat{M}_{ij}^S : -31.5\%$ $\hat{M}_{ij}^{P:E} : 76.9\%$ $\hat{M}_{ij}^{P:W} : 131.9\%$ $\hat{M}_{ij}^H : 76.9\%$

Figure 6: Weibull Persistence, No Degradation: $\alpha = 0.5$, $\theta = 1$

Figure Walk-through

Robert L Wolpert

October 26, 2012

1 Introduction

Figure(1) shows two views of the same simulated two-week period, in which fatality occurs at a rate of about one per day and in which weekly searches were made with proficiency $s_{ij} = 30\%$. Carcass persistence is exponentially distributed with mean $t_{ij} = 15$ d, and $\theta = 50\%$ of undiscovered carcasses remain across search intervals (those that do *not* remain might be thought to have decayed). Table 1 shows the arrival time and fate of each carcass.

Upper Figure

Each fatality is shown in the upper “Time line” figure as an “ \times ” mark, followed by a horizontal line that indicates the fate of this particular carcass.

Carcasses eventually removed by scavengers are shown in red, with a red dashed red line extending from a red cross “ $\times - -$ ” marking the fatality to an open circle “ $- - \circ$ ” marking the removal. Vertical position in this plot is another indicator of persistence— points are sorted so the carcasses removed most quickly are at the top, those removed most slowly at the bottom (for more on this see *p. 3* below).

Carcasses eventually discovered in searches are shown as solid black lines, beginning at a black “ $\times -$ ” marking the fatality and ending at one of the weekly search times on days zero, seven or fourteen. Discoveries are marked by black filled circles “ $- \bullet$ ” for “new” carcasses, *i.e.*, those from the search week, while “old” carcass discoveries are marked with crossed squares “ $- \boxtimes$ ”.

Finally, undiscovered carcasses that become undiscoverable are marked by faint dotted blue lines, beginning at a blue cross “ $\times \dots$ ” marking the fatality and ending unceremoniously at a search time. We’ll discuss the curved lines in the top figure below on *p. 3*.

Lower Figure

The ground “Carcass Count” is shown in the lower figure as a stair-step curve $G(t)$ that indicates the number of discoverable carcasses on the ground at each time t . Between searches, this increases by one with each new fatality and decreases by one with each removal

by scavengers. After each search time, $G(t)$ drops by the number of carcasses discovered and removed by the Search Team, which is indicated by a downward red “ \downarrow ”. In addition, some carcasses may “disappear” as they become undiscoverable (or perhaps decay); if so, $G(t)$ will drop further to begin the next period at a value below the red arrow point, indicated by an open square “ \square ”. The number possibly discoverable at each search is shown by the filled square “ \blacksquare ”.

In the bottom figure, the smooth blue curve shows the expected number of discoverable carcasses for the model assumed by the Pollock and Huso estimators—beginning each period with zero carcasses at a blue “ \circ —”, then rising smoothly over the period to a peak marked with a blue “— \bullet ”, then dropping to zero to begin the next period due to those estimators’ assumption of “zero carcasses beginning each period”, or “no old carcasses”. Those curves generally lie below $G(t)$, because their “no old carcass” assumption is false in this simulation, leading estimators \hat{M}_{ij}^P and \hat{M}_{ij}^H of Pollock and Huso to overestimate M_{ij} on average.

A Walk Through This Simulation

This simulation begins at time $t = 0$ with $G(0+) = 2$ discoverable carcasses present, the remnants of the arrivals, removals, and weekly searches from 50 earlier simulated days (not shown) generated to ensure that this two-week period would be typical. Sixteen additional simulated fatalities occurred between days 0 and 14, about what one would expect for an average daily mortality of $m_{ij} = 1/d$.

The first new fatality occurs 0.838 days (20 hours, 7.5 minutes) into the simulation, indicated by a red \times at the top left in the upper figure and by the unit increase of $G(t)$ by one (from 2 to 3) in the lower one. The top figure shows that this carcass is eventually removed by scavengers at time $t = 2.015$; this event is indicated in the lower curve by a drop of $G(t)$ from 5 to 4.

$G(t)$ had risen to 5 by time $t = 2.015$ due to the second and third fatalities, which arrived just 41 minutes apart at times $t = 1.27$ and $t = 1.30$, increasing $G(t)$ by one at each event. The earlier of these two is eventually removed by scavengers at time $t = 4.92$, but the latter lasts long enough to be discovered by the Search Team on day seven.

The Day 7 Search

The lower figure shows that $G(7) = 8$ discoverable carcasses were present for the day-7 search, and that three were discovered then (because the red arrow “ \downarrow ” extends from 8 down to 5). Two of the three discovered carcasses were “new” ones, that arrived at times $t = 1.30$ and $t = 2.41$; the other one was an “old” carcass, that arrived at time $t = -0.17$, four hours and five minutes before the start of our two week-long simulation. Of the five carcasses that were present but not discovered in the day-7 search, two became undiscoverable (on average we would expect $(1 - \theta) = 50\%$ of them to do so), leaving $G(7+) = 3$ discoverable carcasses just after the search to begin the second week.

In the top figure, the two carcasses that become undiscoverable are indicated by blue “ $\times \dots$ ” marks beginning at times $t = 3.58$ and $t = 5.90$, and ending with the search at

$t = 7$. That figure also reveals the eventual fate of the other $G(7+) = 3$ carcasses that were present but not discovered in the day-7 search— one of them (the one that arrived at $t = 2.42$) is eventually removed by scavengers at time $t = 13.78$, just before the day-14 search, while the other two eventually became undiscoverable (decayed, perhaps), one on day 14 and one later (on day 21, as it happens).

The Day 14 Search

The search on Day 14 discovered three carcasses, all “new” (having arrived at times $t = 10.50$, 11.77 , and 13.23 , all in the range $(7, 14]$). Four carcasses were missed in this search: two that arrived just minutes apart at $t \approx 1.43$, which were also both missed in the search on day 7 and both of which are eventually lost to decay, one arriving at $t = 13.23$, also lost to decay; and one (the red $\times - -$) arriving at $t = 12.58$, that will eventually be removed by scavengers at time $t = 27.13$ after eluding discovery in both the day-24 and day-21 searches.

The Curves in the Top Figure

Height in the top figure is in fact the “quantile” of the persistence times— so half the arrivals (all marked by \times 's) are in the upper half of the figure, 10% in the top (or bottom) tenth, and so on. In fact, the sixteen arrival marks “ \times ” are distributed perfectly evenly (or “uniformly”) in the two-dimensional rectangle with height $0 < y < 1$ and width $0 < t < 14$.

The smooth black curves in the upper figure mark the earliest time a carcass can arrive and still be unremoved by scavengers at the next search time. SO, *every* \times outside all the triangular regions marks the arrival of a carcass that will be removed by scavengers before the next search (and so is red), while *every* “ \times ” inside the triangular regions will still be on the ground at the time of at least one search. If it is undiscovered in that search then it still might be removed by scavengers or to decay (and hence some of those marks are red \times or blue \times). More frequent searches (smaller values of I_{ij} , here 7) reduce loss to scavenging precisely because they reduce the area outside these triangular regions, but evidently there is a rapidly diminishing return on investment when I_{ij} is reduced far below t_{ij} (here 15 d), because there is little remaining area outside the union of triangles; see Figure 2.

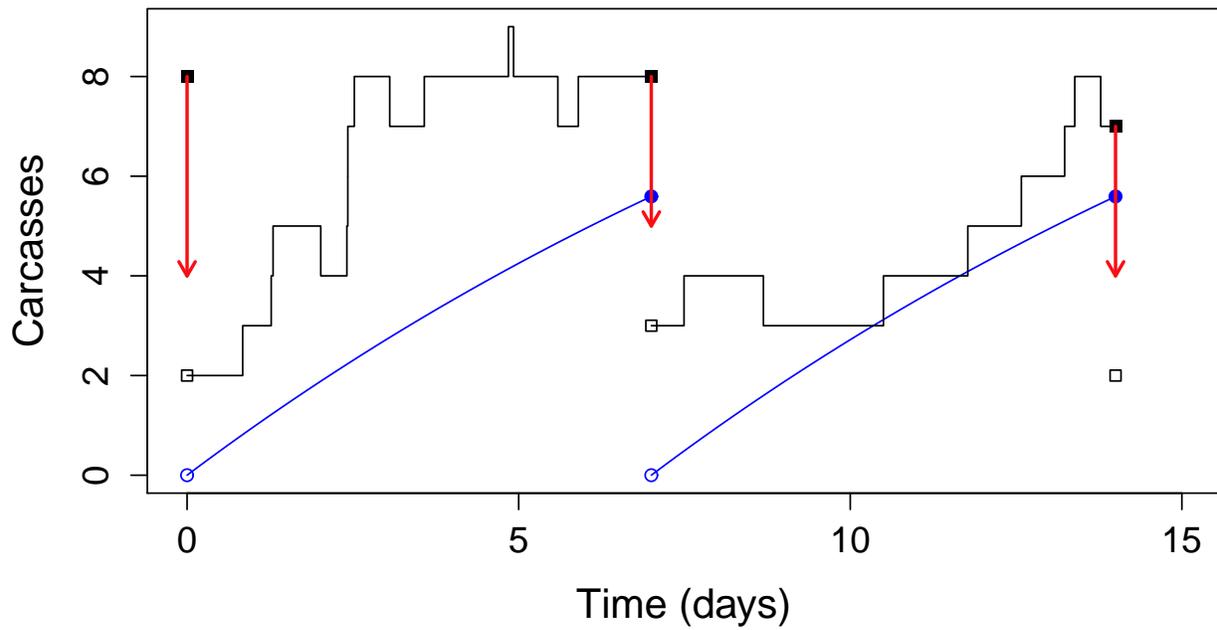
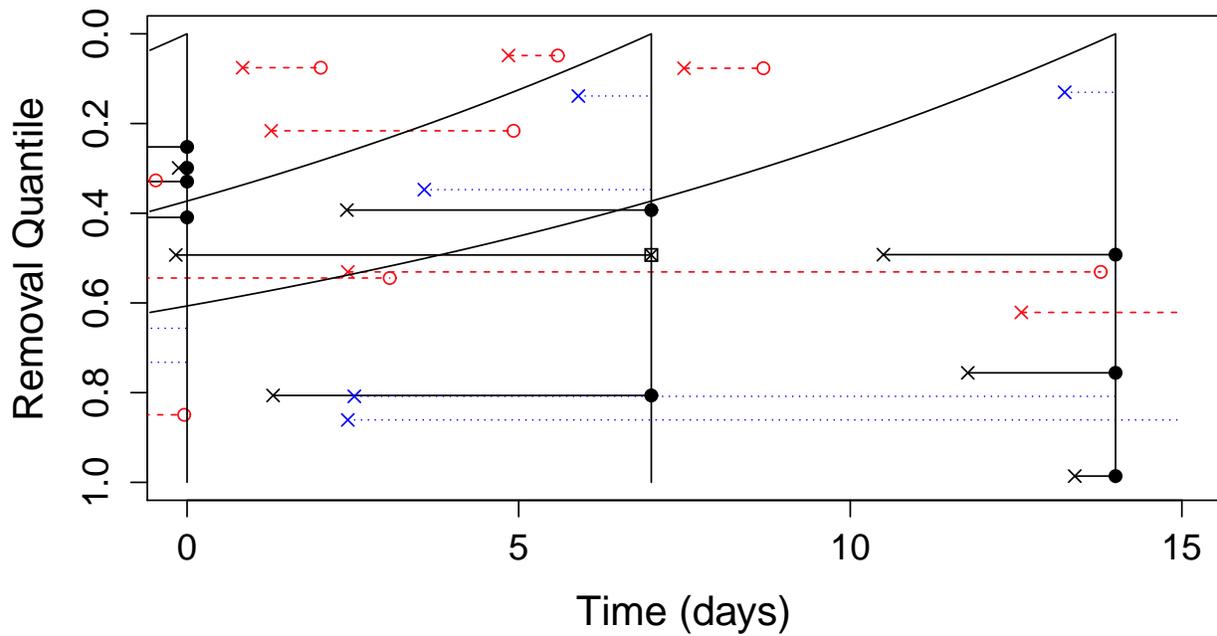


Figure 1: Two graphical views of consecutive one-week searches: Individual time-lines (top), Ground carcass count (bottom). Search proficiency is $s_{ij} = 30\%$; persistence is exponential with mean $t_{ij} = 15$ d; $\theta = 50\%$ of undiscovered carcasses remain discoverable for future searches.

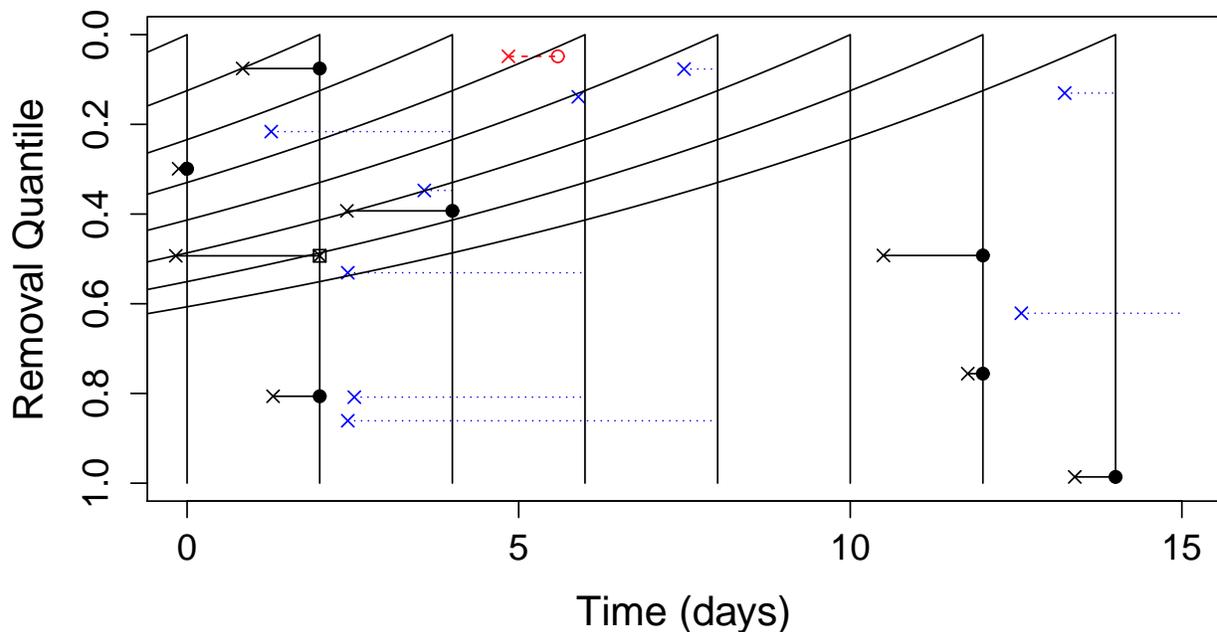


Figure 2: The same simulation, but with $I_{ij} = 2$ -day search intervals. Note fewer carcasses lost to scavenging, but only one more discovery (7 in the seven searches on days 2,4,...,14).

Serial	Arrival	Departure	Fate
	-0.1703770	7	Disc
	-0.1270495	5.201678	Rem
1	0.8383745	2.015476	Rem
2	1.2684557	4.922724	Rem
3	1.2967885	7	Disc
4	2.4092051	7	Disc
5	2.4233033	13.776822	Rem
6	2.4236632	21	
7	2.5218538	14	
8	3.5768155	7	
9	4.8454552	5.590141	Rem
10	5.8996038	7	
11	7.4934336	8.690271	Rem
12	10.5000953	14	Disc
13	11.7721292	14	Disc
14	12.5795863	27.139489	Rem
15	13.2330163	14	
16	13.3854000	14	Disc

Table 1: Arrival and departure times for the sixteen carcasses appearing during period $(0, 14]$ and the two earlier carcasses still present past time $t = 0$.

APPENDIX E:

Public Webinar Presentation

Improving Methods for Estimating Fatality of Birds and Bats at Wind Energy Facilities

California Wind Energy Association Public Webinar

September 26, 2012

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California Energy Commission Grant # PIR-08-028
U.S. Fish and Wildlife Service Grant #13410BG006



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CalWEA

Background

- ✦ 2007 CEC/CDFG Guidelines
 - *Guidelines for Reducing Bird & Bat Impacts from Wind Energy Development*
- ✦ 2008 CEC Research “Roadmap” on Impact Assessment Methods
- ✦ 2008 CEC PIER RFP
- ✦ 2009 CEC PIER Award to CalWEA
 - *Address Guidelines’ Appendix F*
- ✦ 2011 Supplemental FWS Grant to CalWEA

3



CalWEA

Project Goals

- ✦ Improve the accuracy of methods for estimating the number of bird and bat fatalities associated with wind energy facilities
- ✦ Provide guidance leading to improved procedures for mortality monitoring at wind energy facilities

4



Preview of Conclusions

- ✦ Fatality estimators in use often produce biased results
- ✦ This calls into question the appropriate use of traditional estimators where the error would be of consequence, whether for project-specific results, industry averages, or industry totals
- ✦ Standardized methods are needed to generate fatality detection probabilities and fatality estimates
- ✦ Our proposed new estimator produces unbiased results, and requires new field protocols

5



Field Study Design and Findings

Field components:

1. Placement of carcasses (birds and bats) at study turbine strings by Project Field Managers (PFM)
2. Blind carcass searches of strings by Field Technicians (FT)
3. Status checks of placed carcasses by PFM

Findings:

1. Searcher Proficiency
2. Persistence Probability

6



Field Study Design Details

- ✦ In all cases, prior to searches the true number and location of carcasses is known to PFMs, but not to FTs
- ✦ Each string is searched for up to 60 days, or until all carcasses are removed
- ✦ Strings selected to represent various environmental conditions, including grass height, slope, vegetation type
- ✦ Carcasses are tagged and followed consistently throughout study period by PFMs



7



Survey Design Characteristics

- ✦ January 7, 2011 – April 1, 2011
- ✦ Weekly searches by FTs
- ✦ PFMs sampled and noted carcasses approx. every 3 days
- ✦ Blocks of strings sampled simultaneously, surrogate for time changes in ecology

Small bird carcasses placed during study	Bat carcasses placed during study	Incidentally found carcasses added to study	Study length (days)
90	78	21	113

8



CalWEA

FT conducting a search



Field Technician

9



CalWEA

Searching in tall grass



PFM Status Check

10



Searching in short grass



PFM Status Check

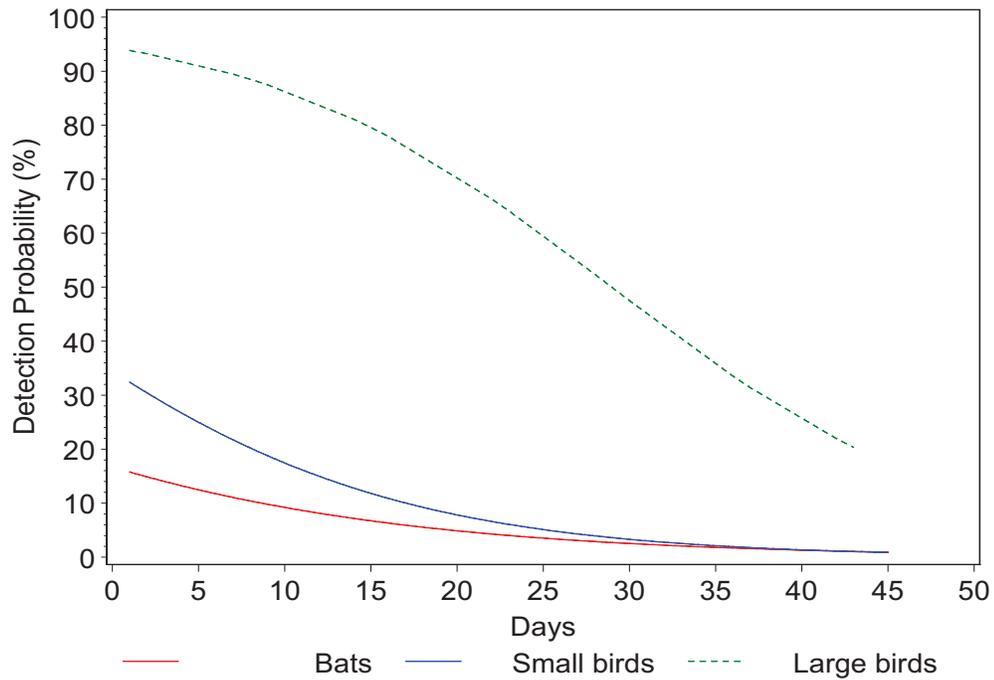


Percent of Birds and Bats Observed

Species	Detected 1 st observation	Average detected over all trials for all observers	Unique carcasses detected during study
Bats	14.1%	8.1%	19.2%
Small Birds	22.2%	17.0%	30.8%
Large Birds	83.3%	67.7%	100%



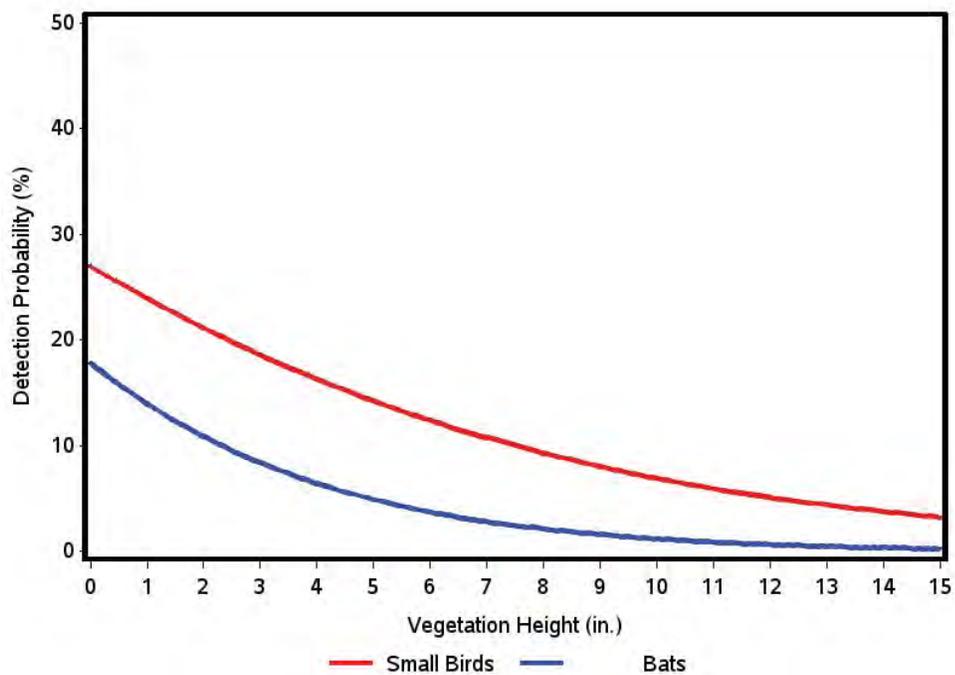
Searcher Proficiency: A Time Dependent Process



13



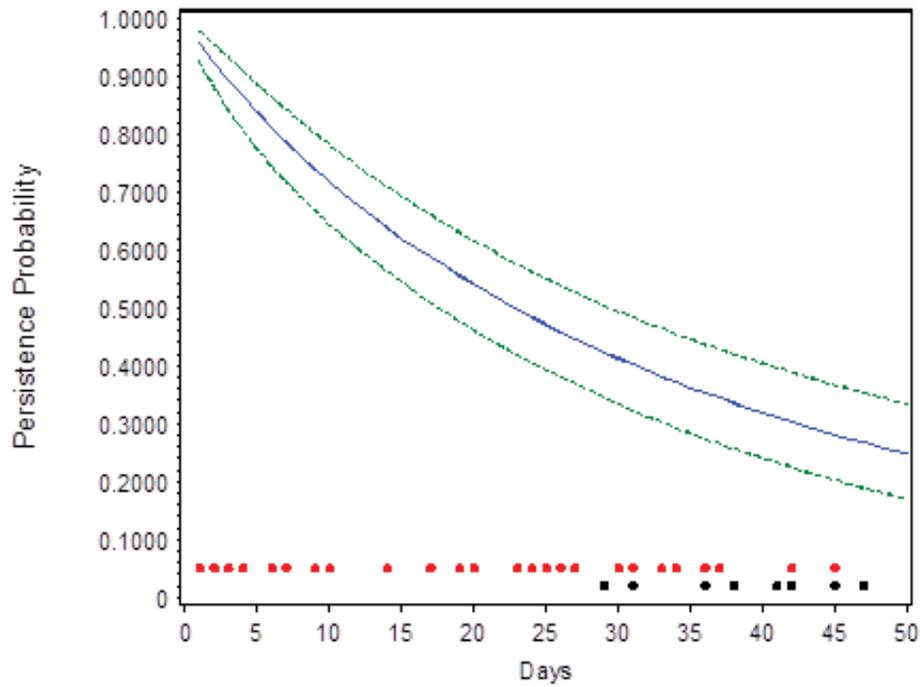
Searcher Proficiency: Dependency on Grass Height



14



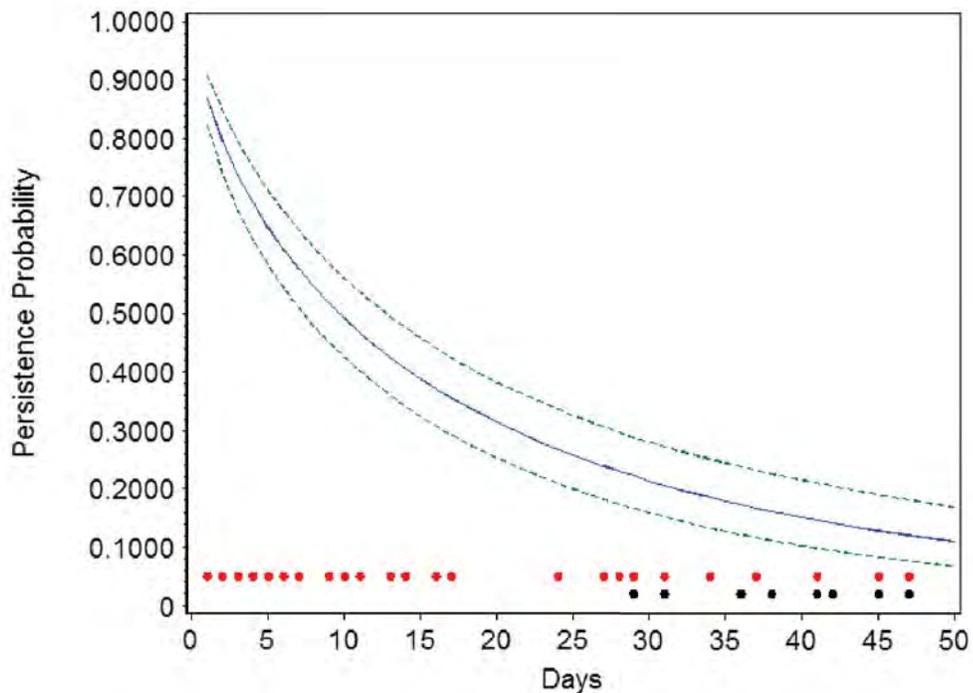
Persistence Probability: Bats (Weibull Distribution, Mean = 43 days)



15



Persistence Probability: Small Birds (Weibull Distribution, Mean = 30 days)



16

Carcass Counts and Avian Mortality

Let's start with **Repeated carcass counts** by Field Technicians at **regular intervals** in **specified regions** near specific turbines, with:

I_{ij} (interval) = days between successive searches,

M_{ij} (mortality) = number of carcasses during interval,

C_{ij} (count) = number of carcasses counted by FTs,

Naïve estimate " $\hat{M}_{ij} = C_{ij}$ " would be okay if:

- A_1 : No carcasses at start of interval;
- A_2 : Every fatality leads to a carcass;
- A_3 : No other carcass sources;
- A_4 : Carcasses remain throughout period;
- A_5 : Field Technicians find every one.

More realistically...

Several authors have proposed improved estimators to accommodate removal by scavengers and discovery failure, based on one or more of:

p_{ij} (persistence) = probability a carcass is unremoved,

r_{ij} (removal rate) = probability per day of scavenging,

s_{ij} (search proficiency) = discovery probability by FTs,

t_{ij} (persistence time) = mean days carcass unremoved.

Each estimator embodies some assumptions.

Erickson & Johnson

Erickson, Strickland, Johnson, Kern (1998):

$$\hat{M}_{ij}^{EJ} = \frac{C_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij}}$$

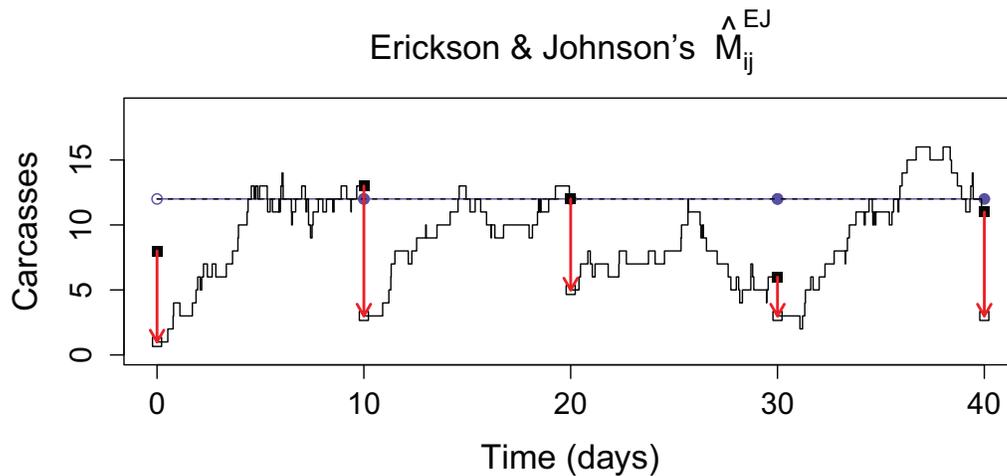
Based on:

$$E[C_{ij}] = M_{ij} (t_{ij} / I_{ij}) (s_{ij})$$

Assumes:

A_1^{EJ} : The system is in equilibrium at each search.

Erickson & Johnson in Pictures



Shoenfeld's Periodic Equation

Shoenfeld (2004):

$$\hat{M}_{ij}^S = \frac{C_{ij} l_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left[\frac{e^{l_{ij}/\hat{t}_{ij}} - 1 + \hat{s}_{ij}}{e^{l_{ij}/\hat{t}_{ij}} - 1} \right]$$

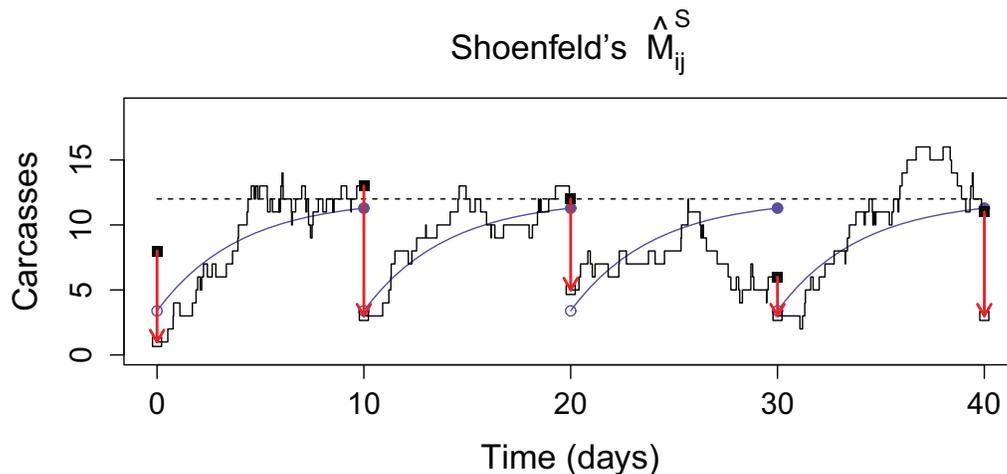
Based on:

$$E[C_{ij}] = \underbrace{M_{ij} (t_{ij}/l_{ij})(1 - e^{-l_{ij}/t_{ij}}) s_{ij}}_{\text{New}} + \underbrace{E[C_{ij}] (1 - s_{ij}) e^{-l_{ij}/t_{ij}}}_{\text{Old}}$$

Assumes:

- A_1^S : Carcass persistence times have exponential distributions.
- A_2^S : **New** & **Old** carcasses have same discovery probability s_{ij}
- A_3^S : Intervals l_{ij} , mortality and removal rates m_{ij} , r_{ij} , and persistence s_{ij} are similar in consecutive intervals.

Shoenfeld in Pictures



Reflection...

- Both \hat{M}_{ij}^{EJ} and \hat{M}_{ij}^S assume “100% bleed-through” — **Every carcass** that is not removed by scavengers, and is not discovered and removed in a search, **remains for** possible discovery in **later searches**.
- Shoenfeld also assumes **exponential distributions** for removal.
- Erickson & Johnson also assume **equilibrium**, even though FTs remove carcasses discovered in searches.
- Always $\hat{M}_{ij}^{EJ} < \hat{M}_{ij}^S$, with small differences if $l_{ij} \gg t_{ij}$:

$$\hat{M}_{ij}^{EJ} < \hat{M}_{ij}^S = \hat{M}_{ij}^{EJ} \left[1 + \frac{s_{ij}}{e^{l_{ij}/t_{ij}} - 1} \right]$$

Estimators without bleed-through

Perhaps **none** of the “Old” carcasses that were

- Not removed by scavengers, and
- Not discovered & removed by FTs

are **ever discovered** in subsequent searches.

If so, we can **remove the bleed-through part** from Shoenfeld’s equation to get a new estimator:

Huso's Equation

Huso (2011) proposed:

$$\hat{M}_{ij}^H = \begin{cases} \frac{C_{ij} l_{ij}}{\hat{s}_{ij} \hat{t}_{ij} (1 - e^{-l_{ij}/\hat{t}_{ij}})} & l_{ij} < \hat{l}_{ij} \\ \frac{C_{ij} l_{ij}}{\hat{s}_{ij} \hat{t}_{ij} (1 - e^{-\hat{l}_{ij}/\hat{t}_{ij}})} & l_{ij} > \hat{l}_{ij} \end{cases}$$

Based on:

$$E[C_{ij}] = M_{ij} (t_{ij}/l_{ij})(1 - e^{-l_{ij}/t_{ij}}) s_{ij}$$

Assumes:

- A_1^H : Each period begins with no discoverable carcasses.
 A_2^H : Persistence times have exponential distributions.

Huso's Equation

Huso (2011) proposed:

$$\hat{M}_{ij}^H = \frac{C_{ij} l_{ij}}{\hat{s}_{ij} \hat{t}_{ij} [0.99 \wedge (1 - e^{-l_{ij}/\hat{t}_{ij}})]}$$

Based on:

$$E[C_{ij}] = M_{ij} (t_{ij}/l_{ij})(1 - e^{-l_{ij}/t_{ij}}) s_{ij}$$

Assumes:

- A_1^H : Each period begins with no discoverable carcasses.
 A_2^H : Persistence times have exponential distributions.

Huso's Equation

Huso (2011) proposed a slight variation on:

$$\hat{M}_{ij}^H \approx \frac{C_{ij} I_{ij}}{\hat{s}_{ij} \hat{t}_{ij} (1 - e^{-I_{ij}/\hat{t}_{ij}})}$$

Based on:

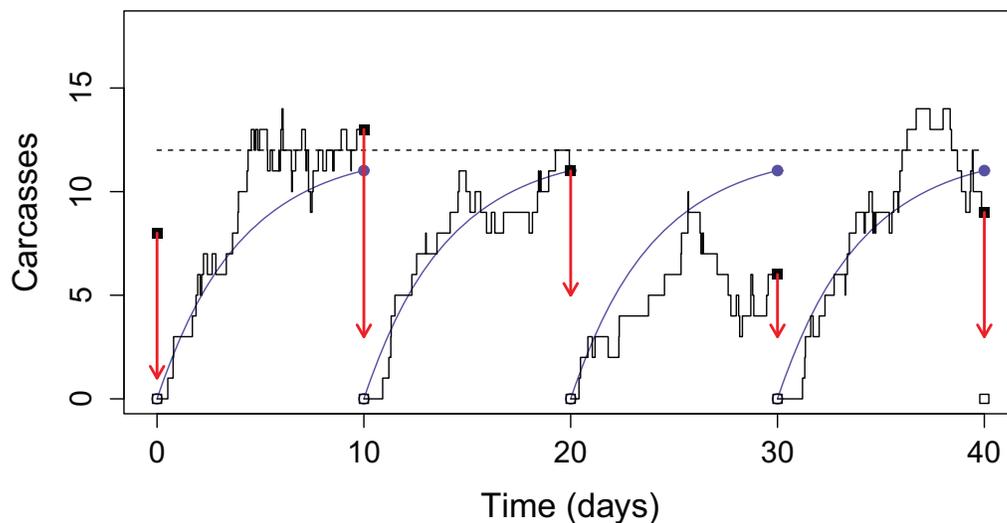
$$E[C_{ij}] = M_{ij} (t_{ij}/I_{ij})(1 - e^{-I_{ij}/t_{ij}}) s_{ij}$$

Assumes:

- A_1^H : Each period begins with no discoverable carcasses.
- A_2^H : Persistence times have exponential distributions.

Huso in Pictures

Huso's \hat{M}_{ij}^H (no bleed-through)



Pollock's Estimator

Pollock (2007) based an estimator on “average probability a carcass is unremoved until the search” p_{ij} , instead of more commonly used mean persistence time t_{ij} :

$$\begin{aligned}\hat{M}_{ij}^P &= \frac{C_{ij}}{\hat{s}_{ij} \hat{p}_{ij}} && \text{(any dist'n)} \\ &= \frac{C_{ij} l_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} [1 - e^{-l_{ij}/\hat{t}_{ij}}]^{-1} && \text{(w/Expon.)} \\ &= \frac{C_{ij} l_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} P\left(\frac{1}{\alpha}, [\Gamma(1 + \frac{1}{\alpha}) l_{ij}/\hat{t}_{ij}]^\alpha\right)^{-1} && \text{(w/Weibull)}\end{aligned}$$

Based on:

$$E[C_{ij}] = M_{ij} \times p_{ij} \times s_{ij}$$

Assumes:

A_1^P : Each period begins with no discoverable carcasses.

More reflection...

- Both \hat{M}_{ij}^H and \hat{M}_{ij}^P assume “0% bleed-through” — “Old” carcasses are never discovered.
- Huso also assumes exponential distributions for removal.
- Pollock's equation can be instantiated for any specific removal distribution, or used in “raw” form with direct empirical estimates of \hat{p}_{ij} .
- For exponential removal,

$$\hat{M}_{ij}^{EJ} < \hat{M}_{ij}^S < \hat{M}_{ij}^P = \hat{M}_{ij}^H$$

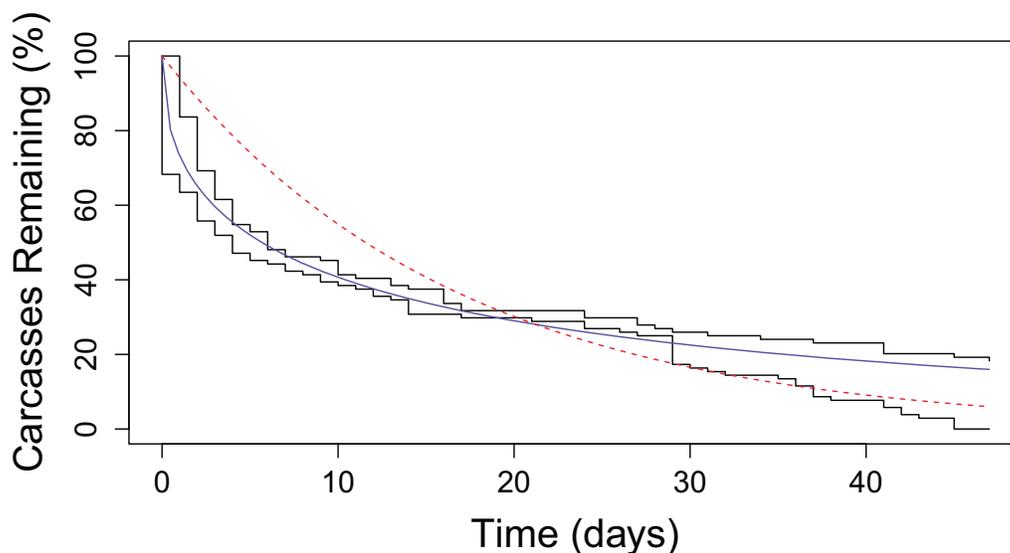
(unless $l_{ij} > 4.6t_{ij}$, when \hat{M}_{ij}^H is 1% bigger than \hat{M}_{ij}^P)

Do the differences matter?

- If $l_{ij} \gg t_{ij}$ (That is, if search intervals are long compared to mean persistence times), Then NO, all four estimators give about the same answers.
- BUT, they differ substantially under more frequent sampling.
- For typical search proficiencies of $25\% < s_{ij} < 60\%$,
 - If $l_{ij} \approx 2t_{ij}$, \hat{M}_{ij}^{EJ} is 4–9% lower than \hat{M}_{ij}^S
 - $\hat{M}_{ij}^H = \hat{M}_{ij}^P$ is 6–11% higher;
 - If $l_{ij} \approx t_{ij}$, \hat{M}_{ij}^{EJ} is 13–26% lower than \hat{M}_{ij}^S
 - $\hat{M}_{ij}^H = \hat{M}_{ij}^P$ is 17–38% higher;
 - If $l_{ij} \approx \frac{1}{2}t_{ij}$, \hat{M}_{ij}^{EJ} is 28–48% lower than \hat{M}_{ij}^S
 - $\hat{M}_{ij}^H = \hat{M}_{ij}^P$ is 32–83% higher
- No matter how short l_{ij} is, \hat{M}_{ij}^S is never more than s_{ij} times smaller than $\hat{M}_{ij}^H = \hat{M}_{ij}^P$ (at most a factor of 3 or 4).

Are the Assumptions True?

Exponential persistence?



Are the Assumptions True?

Exponential persistence?

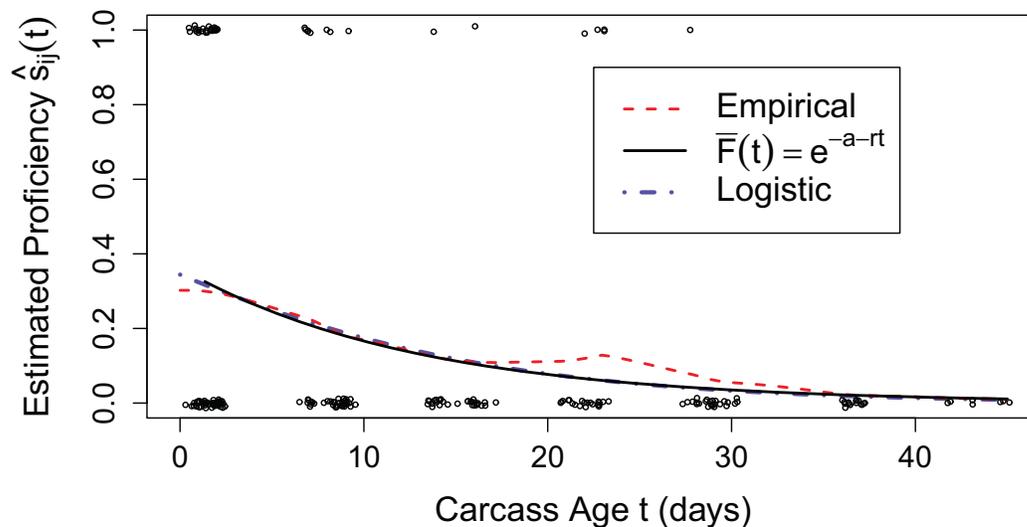
- Weibull, Lognormal, Fisk (Log-logistic) distributions *all* fit data far better than Exponential.
- $Ex(\lambda)$ is the special case “ $\alpha = 1$ ” of $We(\alpha, \lambda)$
- Maximum Likelihood Estimate of α for small birds was

$$\hat{\alpha} = 0.4606, \text{ Standard Error} = 0.0532$$

- MLE is 10.14 SEs away from Exponential Dist'n ($\alpha = 1$)... P -value is 1.8×10^{-24} , far below 0.05.
- Higgs Boson discovery claim was based on 5 SE difference.

Are the Assumptions True?

Constant Proficiency s_{ij} ?



Zero bleed-through? 100% bleed-through?

- For long search intervals, bleed-through doesn't matter.
- For short search intervals, probably some old carcasses are discovered.
- Falling search proficiency with carcass age ameliorates this—old carcasses are less likely to be found.
- **None** of the existing estimators **reflects falling s_{ij}** .

A note about “Mean Persistence Time” t_{ij}

- Most estimators depend on estimates of average time t_{ij} until removal by scavengers.
- This is a **hard** thing to estimate, because of **censored observations** and **heavy tails**.
- If persistence really had **exponential distributions** it wouldn't matter... but evidence shows that's **wrong**.
- **Methods** recommended in the **literature** systematically **underestimate t_{ij}** .
- The relation between daily removal rates and t_{ij} is **highly model-dependent**.

Something New: A Unified Partially-Periodic Estimator

$$\hat{M}_{ij}^* = \frac{C_{ij} l_{ij}}{\hat{s}_{ij} \hat{t}_{ij}} \left\{ \frac{e^{l_{ij}/\hat{t}_{ij}} - \theta(1 - \hat{s}_{ij})}{e^{l_{ij}/\hat{t}_{ij}} - 1} \right\},$$

with “bleed-through” parameter $0 \leq \theta \leq 1$. Special cases:

- $\theta = 0$: Identical to estimator of Huso and Pollock
- $\theta = 1$: Identical to estimator of Shoenfeld
- $\theta = \frac{1}{1-s_{ij}}$: Identical to estimator of Erickson & Johnson

For values $0 < \theta < 1$, this interpolates among them.

Based on:

$$E[C_{ij}] = M_{ij} (t_{ij}/l_{ij})(1 - e^{-l_{ij}/t_{ij}}) s_{ij} + \theta E[C_{ij}] (1 - s_{ij}) e^{-l_{ij}/t_{ij}}$$

But...

- This form of the **Partially Periodic Estimator** allows one to overcome the bleed-through problem, **but**
- It still assumes **exponential distributions** (as do Shoenfeld's and Huso's estimators)
- It still assumes **constant search proficiency** (as do all four previous estimators)
- **Something better** is needed.

Something Better

A new estimator based on the assumptions:

- **Declining removal rate** achieved through Weibull $We(\alpha, r)$ persistence distribution

$$P[\tau > t] = e^{-(rt)^\alpha}$$

for some parameters $0 < \alpha \leq 1, r > 0$;

- **Diminishing proficiency**

$$s_{ij}(t_k) = e^{-a-bt_k}$$

where t_k denotes the “age” of k th carcass

- **Partial periodicity**, with a fraction $0 \leq \theta \leq 1$ of remaining carcasses still discoverable
- Parameters $(\alpha, r), (a, b), \theta$ are estimated in **Detection Probability Trials** designed to accommodate censoring.

The New Estimator:

$$\hat{M}_{ij}^* = \frac{C_{ij} l_{ij}}{R_{ij}^*}$$

where C_{ij} is the Carcass Count, l_{ij} is Interval Length and, for $\theta = 0$ (no bleed-through, as in Huso and Pollock), and R_{ij}^* is the *adjusted* (for diminished search proficiency) *remainder factor*:

$$R_{ij}^* = \int_0^{l_{ij}} \exp\left(- (r_{ij} x)^\alpha - a - bx\right) dx$$

Easily calculated on a computer— or, for $\alpha \approx 1/2$ as in our small bird data, available explicitly as

$$R_{ij}^* = Q^*(0; a, b, r_{ij}) - Q^*(l_{ij}; a, b, r_{ij}), \quad \text{where}$$

$$Q^*(x; a, b, r) = \exp\left(-a - bx - \sqrt{rx}\right) / b$$

$$- e^{-a+r/4b} \Phi\left(-\sqrt{2bx} - \sqrt{r/2b}\right) \sqrt{\pi r/b^3}.$$

Summary

New estimator:

- Includes as special cases those of Erickson & Johnson, Shoenfeld, Huso, and Pollock,
- Extends to **Partially Periodic** case of $\theta > 0$,
- Allows **exponential** or more realistic **Weibull** persistence times with **declining removal rate**,
- Allows **constant** or more realistic **diminishing proficiency**,
- Is not much harder to use than existing ones, after an **integrated detection trial** to estimate the five parameters
 - (α, r) (for persistence distribution),
 - (a, b) (for proficiency), and
 - θ (for bleed-through).

Suggestions:

- For short intervals $I_{ij} \ll t_{ij}$, **bleed-through** θ is important—getting it wrong will distort estimates (up or down).
- For long intervals $I_{ij} \gg t_{ij}$, **diminishing proficiency** $s_{ij}(t)$ is important. So is **declining removal rate**: use Weibull or Log Normal or Log Logistic removal distributions, *not* exponential.
- For moderate intervals $I_{ij} \approx t_{ij}$ and search proficiencies s_{ij} , ratios among estimators are no worse than about 3:2.
- All these issues (**bleed-through**, **diminishing proficiency**, **declining removal rate**) can be addressed with a modest increase in complexity with a **partially periodic estimator** and a suitable **integrated detection probability trial**.



Summary: Model Comparison

Model Characteristics

- ✦ Contrasting with lessons from the field work:
 - All models assume constant searcher proficiency
 - Some models assume an exponential distribution (fresh and older carcasses equally attractive to scavengers)
- ✦ Some models assume bleed-through (Shoenfeld), some don't (Huso, Pollock), and E&J assume equilibrium

19



Summary: Model Comparison

- ✦ For exponential removal:
 - Erickson & Johnson < Shoenfeld < Pollock < Huso**
- ✦ Even though biased, if search interval is long compared to mean persistence time:
 - **All 4 estimators give about the same results**
- ✦ But, if search interval is short relative to persistence:
 - **Differences among equations increase**

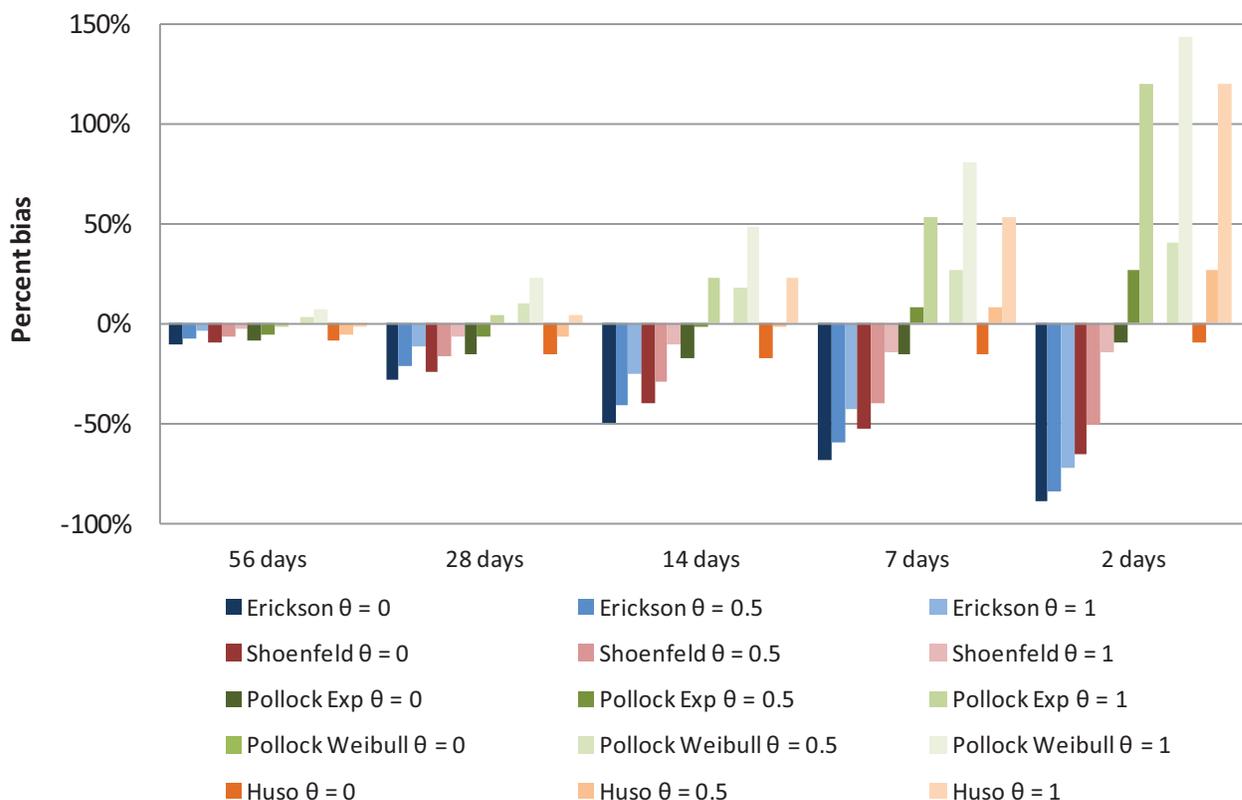
20



Are Short Search Intervals Useful?

- ✧ Short search intervals increase chance of bias
 - Short intervals do not allow system to reach equilibrium, inconsistent with E&J assumption
 - Huso and Pollock assume 0% bleed-through, therefore bias will occur if bleed-through is more
 - Shoenfeld assumes 100% bleed-through, therefore bias will occur if bleed-through is less
- ✧ New partially-periodic equation allows for any bleed-through, therefore works very well with short or long intervals

Comparison of Bias in Estimators at Various Search Intervals and "Bleed-Through" θ Assumptions with Removal Distribution $\alpha = 0.7$





Recommendations

Given the shortcomings of traditional estimators ...

Traditional fatality estimators do not sufficiently account for

- Time-dependent processes of carcass persistence and searcher proficiency, and
- “Bleed-through” (the portion of carcasses persisting through a search interval that can be detected in subsequent search interval)

... CalWEA’s Research Team developed and recommends:

- New fatality estimator (“partially-periodic” presented above), and
- Integrated detection probability trial methodology

23



Why Traditional Detection Trials Won't Work

- ✦ Traditional Searcher Proficiency Trials
 - Only fresh carcass detection events
 - One day trials
- ✦ Traditional Carcass Persistence Trials
 - No way of measuring bleed-through

24



Requirements for a New Integrated Detection Probability Trial

1. A preliminary traditional carcass persistence trial
2. Strategic placement of trial carcasses
3. Traditional schedule of carcass checks, with additional checks on the same day as scheduled searches
4. Searchers record detected trial carcasses over multiple search intervals
5. Measure the proportion of carcasses that persist (bleed-through) from one search interval to the next to derive the term θ

25



Analytical Products Gained From New Integrated Detection Trial

1. Time dependent probabilities for carcass persistence and searcher proficiency
2. A measurement of θ (bleed-through)
3. Traditional fatality estimator parameters are conserved

26



Conclusions

Policy Decision Implications

- ✦ Potentially faulty fatality estimates are being used in decision-making
- ✦ Are the errors of consequence?
 - Accuracy vs. precision
- ✦ Caution is required ...

27



Conclusions

Policy Decision Implications

- ✦ Caution needed in determining:
 - National avian and bat mortality
 - Industry averages
 - Regulatory standards for monitoring
 - Numerical “thresholds” for post-construction monitoring and adaptive management requirements
- ✦ Caution needed when comparing:
 - Specific project results to national industry averages
 - Intra-project results where study approaches have differed
 - Results among wind facilities
- ✦ What degree of accuracy and precision is needed?

28



Conclusions

Study Design Implications

- ✦ To generate accurate and comparable fatality detection probabilities and fatality estimates
 - Uniform, standardized methods are needed
 - Partially periodic equation produces unbiased results
 - New equation requires new field study protocols

29

Final Thoughts & Questions

This webinar will be posted (early October)
and the final report by (November)

www.calwea.org

